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RESPONSE THEORY FOR ALLUVIAL RIVER ADJUSTMENTS TO ENVIRONMENTAL AND MAN-MADE CHANGES

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A response theory is developed for prediction of the direction and magnitude of alluvial river adjustments from one regime to another due to environmental and man-made changes. The theory makes use of the tendency of alluvial channels toward dynamic equilibrium after being disturbed by an extreme event. Various extremal concepts are adopted such as energy dissipation (including its special cases of stream power, unit stream power and energy slope), sediment efficiency, friction factor and Froude number. The regime theory hydraulic exponents, which are usually taken as constants, are derived in a more general fashion where the classical reported values could be obtained. After selecting the function that describes each concept and setting its variation to zero, a equation that represents the response of the river channel is obtained. This response is given in the form of a relation between adjustments or variations between the width, depth, slope, discharge, and channel roughness. Comparisons with field data for ten cases of river response show the success and potential of this approach.

INTRODUCTION

With the advent of the new millennium many advancements have been accomplished in engineering sciences. However, the problem of predicting alluvial river channel response to environmental and man-made changes still awaits a reliable, direct, and straightforward method. Environmental changes include climatic, geologic, hydrologic, and hydraulic changes, while man-made changes include dam construction. As far as rivers are concerned, all these factors have in common that they change the inflow discharge of water and sediment carried down to the river channel.

Streams have a tendency to adjust to these changing conditions by adjusting their hydraulic geometry. Adjustments in hydraulic geometry are accomplished by adjustments in channel depth, width, velocity, and longitudinal profile (slope) in order to accommodate the transport of water and sediment within the constraints imposed. Changes in river regime or hydraulic geometry are characterized by: (1) the direction of change (e.g. increase or decrease), (2) the magnitude of change and, (3) the time required for the change to occur from one regime to another. Herein, the first two factors are addressed. For management, planning, design, and operation of rivers and open channels, predicting channel hydraulic geometry response is an essential engineering task.

Lane (1955) introduced an interesting relation that describes adjustments in the channel regime due to natural or man-made changes. The relation is given as

$$QS \approx Q_S D_{50} \tag{1}$$

where Q is the channel discharge, S is the channel slope, Q_s is the sediment load, S is the channel slope, and D_{50} is the diameter of which 50 percent of the bed material is finer. This equation is a useful tool in determining the direction of adjustments but not the magnitude, i.e., it is a qualitative, but not a quantitative tool.

Existing hydraulic geometry prediction methods can be divided into three major categories: (1) regime methods, (2) extremal methods and (3) stability methods. In regime methods, channel width, depth, and slope are given as a function of channel forming discharge, often taken as the bankfull discharge. Such relations were based on canals and rivers assumed to be in regime or equilibrium in India, Pakistan, and the United States. The general forms of these relations are

$$B = aQ^b \tag{2}$$

$$D = cQ^f \tag{3}$$

$$V = kQ^m \tag{4}$$

$$S = iQ^j \tag{5}$$

in which B is the channel top width, D is the average depth, V is the average velocity, and a, b, c, f, i, j, k, and m are empirical coefficients. Leopold and Maddock, 1953, using 20 river cross-sections (at-a-station) reported mean values of "*b*" as 0.26, "*f*" as 0.4 and "*m*" as 0.34. Lacey (1930, 1958), using data form canals assumed in regime in India and Pakistan, determined "*b*" as 0.5 and "*j*" as -1/6. Blench (1952, 1970) following Lacey's approach, determined "*b*" as 0.5, "*f*" as 1/3, "*m*" as 1/6 and "*j*" as -1/6. Simons and Albertson 1960, using India-Pakistan canals and others in Colorado, Wyoming and Nebraska in USA determined "*b*" as 0.5 and "*f*" as 0.36.

It can be seen that these relations state that the channel has only one width and depth for a given discharge. Chang (1988) states that under the regime relationship the width is a function of discharge, but under transient changes, the channel can have very different widths even though the discharge is essentially uniform along the channel. In addition, the regime equations indicate that the channel width, depth, and velocity are independent of the channel slope.

Milhous (1988), using the regime equation, Equation (2), for channel width, expressed changes of channel width due to changes in discharge by

$$B = B_0 \left(\frac{Q}{Q_0}\right)^b \tag{6}$$

in which B is the new channel width, B_o is the initial width, Q is the new discharge, and Q_o is the initial discharge. The average value of b was taken as 0.54 according to Leopold et al. (1964). Equation (6) was tested through prediction of changes in channel width resulting from dam construction for which data by Williams and Wolman 1984 was assembled. However Equation (6) did not predict the direction of width change in 11 of 16 examples presented.

In an empirical approach similar to the regime theory, Schumm (1969) used data from sand bed rivers in the Great Plains of the United States and New South Wales, Australia. The shape of the channel cross section was related to the percentage of silt-clay content measured from the channel perimeter. The percentage of the silt-clay content in the channel was used as an index for the ratio of bed material load to total load. In this context channel geometry is directly related to sediment load.

Past investigators used extremal hypotheses to explain adjustments in channel geometry. Mackin (1948) stated that the river channel adjusts itself in order to accommodate the transport of debris. Langbein (1964) presented the theory of minimum variance to explain changes of the hydraulic exponents of the regime type equations. The variance was defined as the sum of the square of the hydraulic exponents appearing in Equations 2-5, which is set to a minimum to be consistent with local restrictions. Yang (1987) stated the problem of the selection of the correct combinations of the variables used in the minimization and the fact that different combinations lead to different answers. However, Langbein's concept of minimization inspired research on the application of minimization or maximization hypotheses to explain hydraulic geometry.

Yang (1976) introduced the concept of minimum unit stream power, *VS*. Later, Yang and Song (1979) introduced the theory of minimum rate of energy dissipation and derived the minimum stream power, γQS, as a special case. The theory was applied, Yang et al. (1981), to the study of channel geometry by driving theoretically the hydraulic exponents as $b = f = 9/22$, $m = 4/22$, and $j = -2/11$. They stated that channel depth can readily be adjusted in accordance to the theory, while width adjustment may also depend on constraints other than discharge and sediment load.

Similarly, Chang (1979) employed a hypothesis based on the energy or power approach for determining the channel width of alluvial rivers. Chang (1980, 1985) presents a rational design method to determine the stable width, depth and slope for sand bed canals based on the physical relations of sediment transport, flow resistance, and minimum stream power for a certain bank stability. The method is iterative in the sense that different widths and depths are assumed followed by computation of slope using a bed load equation and then computation of discharge. The procedure is repeated until the computed discharge matches the given discharge. Thus the procedure lends itself to a computer solution because many cases for different combinations of depth and width need to be made. Chang obtained b=0.5, f=0.3 and j=-1/6 for alluvial canals.

Kirkby (1977) used the hypothesis of maximum sediment efficiency that later was used by White et al. (1982). White et al. (1982) and Chang (1988) showed the equivalency between the minimum stream power concept and the maximum sediment efficiency or discharge. Davies and Sutherland (1983a) showed that the conditions at which unit stream power or total stream power is minimized are also those at which friction factor is maximized. The exception is in the short-term adjustment where water discharge Q and S are independent variables and therefore QS cannot be minimized. Rammett (1980) used the concept of maximum Froude number along with dominant discharge and no or saturated sediment discharge. His approach bears resemblance to minimum energy dissipation and maximum sediment discharge. Kondap and Garde (1980), and Hancu and Batuca (1980, 1988) used the variational principle for determining the hydraulic geometry of stable alluvial channels which is reduced to the minimum power or maximum sediment efficiency concepts. Hancu and Batuca (1988) obtained f=0.4, b=0.5 and m=0.1 when the relative roughness (depth/roughness height) exponent is 1/4, and that f=0.37, b=0.56 and m=0.06 when the exponent is 1/6.

Among the stability methods, Parker (1978) attributed the stable width of straight alluvial channels to the balance of lateral sediment exchange between the bank zone and the central region. Lateral sediment exchange was related to the mechanics of internal river flow. Stevens (1989) related channel width of straight alluvial channels and canals with or without a bed-material load to the tractive strength and the sliding strength of the bank soils. Based on these two criteria, he determined more than one width and slope to carry a given water discharge and sediment load, for which the minimum width was selected as the best design criteria. No criteria were set for the maximum channel width where its dependence was attributed to the depositional characteristics of the suspended sediment and the development of meandering tendencies in wider channels.

From the foregoing review it is noted that past researchers have focused on predicting the absolute value of the channel hydraulic geometry corresponding to an average channel regime. Current methods are qualitative or quantitative in the sense of estimating the hydraulic exponents. Qualitative methods give only descriptive information about the direction of change but not the magnitude. Quantitative methods predict the hydraulic exponents as constants without expressing the interrelation between the hydraulic exponent themselves.

Hafez (1999) presented a generalized theory based on a variational approach that qualitatively and quantitatively describes adjustments in the width, depth, and slope of alluvial river channels. The extremal concepts of energy dissipation, friction factor, sediment transport and Froude number were utilized. By setting the variation of the function describing each concept to zero, a relation results between adjustments in the width, depth and slope due to variations of discharge. The generalized theory was so named because it can be applied to any extremal concept in addition to the regime equations. However in reality, the generalized theory functions as a theory of river response. Therefore, it is termed here the response theory. The generalized theory presented in Hafez (1999) was not tested against field data. Application to field data is carried out here and, for the sake of completeness, derivation of the theory is presented in more general form and details than in Hafez (1999).

THE RESPONSE THEORY OF RIVER ADJUSTMENTS

Following Hafez (1999) the following assumptions are used in this analysis: (1) rectangular and one-dimensional channel, (2) straight and wide channel, (3) at-a-station hydraulic geometry analysis, and (4) first order variations of the relevant variables.

The equilibrium conditions of a river channel are a function of channel discharge, roughness, energy slope, width, depth, sediment discharge, and bed material size. In functional form, this can be expressed as:

$$\Psi = Fct.(Q, \Phi, S, B, D, Q_s, D_s) \tag{7}$$

where Ψ is a variable that describes the equilibrium conditions of the river, Φ is a measure of channel roughness, Q_s is the sediment discharge, and D_s is a measure of bed material size. If there is a change in any of the variables in Equation (7), then, according to the response theory, in order to restore equilibrium the corresponding change of Ψ should be zero, i.e.

$$\Delta\Psi = \sum_i \left(\frac{\partial\Psi}{\partial x_i}\Delta x_i + \frac{1}{2}\frac{\partial^2\Psi}{\partial x_i^2}\Delta x_i^2 + \frac{1}{6}\frac{\partial^3\Psi}{\partial x_i^3}\Delta x_i^3 + \text{H.O.T.}\right) = 0$$

or equivalently

$$\Delta\Psi / \Psi = 0 \tag{8}$$

where x_i is any of the variables appearing in Equation (7) and H.O.T. are higher order terms. Equation (8) is the general equation that corresponds to an extremal condition (either maximum or minimum) of the function describing or representing equilibrium conditions of the river channel. Such functions can be the stream power, the friction factor, the sediment transport, or the Froude number. The following shows how Equation 8 is applied to obtain the equations predicting the width, depth, and slope changes. Following the assumption of having only first order variations, only the first term in the right hand side of Equation (8) is considered while the other terms are neglected, i.e:

$$\Delta\Psi = \sum_i \frac{\partial\Psi}{\partial x_i}\Delta x_i = 0 \tag{9}$$

Extremal Friction Factor

For a very wide channel the friction factor at a channel cross section is expressed as

$$f_r = \frac{8gD^3SB^2}{Q^2} \tag{10}$$

where f_r is the Darcy-Weisbach friction factor and g is the gravitational acceleration. Equation (10) viewed in the light of Equation (7), states that the friction factor when considered as representing channel equilibrium has a functional dependence upon the discharge, depth, width, and slope.

For the friction factor to have extremal conditions, the variation of its function is set to zero, i.e. Δf_r=0, which could be expressed as

$$\Delta f_r = \frac{\partial f_r}{\partial D}\Delta D + \frac{\partial f_r}{\partial Q}\Delta Q + \frac{\partial f_r}{\partial B}\Delta B + \frac{\partial f_r}{\partial S}\Delta S = 0 \tag{11}$$

applying Equation (11) to Equation (10) yields

$$\Delta f_r = 8g\{3\frac{S}{Q^2}B^2D^2\Delta D - 2\frac{D^3}{Q^3}SB^2\Delta Q + 2\frac{D^3}{Q^2}BS\Delta B + \frac{D^3}{Q^2}B^2\Delta S\} = 0 \tag{12}$$

Dividing Equation (12) by Equation (10) (equivalent to setting $\Delta f_r/f_r$=0) yields

$$3\frac{\Delta D}{D}-2\frac{\Delta Q}{Q}+2\frac{\Delta B}{B}+\frac{\Delta S}{S}=0 \tag{13}$$

solving for ΔB yields

$$\Delta B = B\{\frac{\Delta Q}{Q}-\frac{3}{2}\frac{\Delta D}{D}-\frac{1}{2}\frac{\Delta S}{S}\} \tag{14}$$

where ΔB=$B2$-$B1$, ΔQ=$Q2$-$Q1$, ΔD=$D2$-$D1$ and ΔS=$S2$-$S1$. Subscript 1 refers to equilibrium conditions before channel disturbance and subscript 2 refers to equilibrium conditions after the disturbance. Though Equations (13) and (14) are equivalent, the form of Equation (14) will be used in the applications that follow. This is due to the nature of applications which focus on predicting width changes, a challenging task so far. It should be noted that the prediction equations for channel width are restricted by bank erodibility, which limits increase of channel width and sediment availability, and in turn limits width reduction. In a similar manner, bed armoring limits bed scour and sediment availability limits bed rising. These constraints are easily implemented when using Equation (14). For example, if the channel has firm banks or no changes in the channel width are expected, the term ΔB is simply set to zero in Equation (14). The same treatment is applied to other terms in Equation (14) such as depth or slope when they are constants.

Extremal Sediment Efficiency

In this approach a sediment transport function is needed. Two equations are selected in this context, namely that of Engelund and Hansen (1967) and simplified Yang (1986) but other equations can be used as well. The Engelund and Hansen (1967) equation is

$$Q_s = K\frac{Q^2 S^{3/2}}{D_{50} B D^{1/2}} \tag{15}$$

where Q_s is the sediment discharge and K is a constant. At extremal conditions ΔQ_s=0 which, when applied to Equation 15 after algebraic manipulation, yields

$$\Delta Q_s = 2\frac{\Delta Q}{Q}-\frac{\Delta D_{50}}{D_{50}}+\frac{3}{2}\frac{\Delta S}{S}-\frac{\Delta B}{B}-\frac{1}{2}\frac{\Delta D}{D}=0 \tag{16}$$

Dividing Equation 16 by Equation 15 and solving for ΔB yields:

$$\Delta B = B\{2\frac{\Delta Q}{Q}-\frac{\Delta D_{50}}{D_{50}}+\frac{3}{2}\frac{\Delta S}{S}-\frac{1}{2}\frac{\Delta D}{D}\} \tag{17}$$

Equation 17 includes the effect of the bed material mean diameter, which is absent in Equation 14.

Yang's simplified form (1986) for the sediment transport function is

$$Q_s = K\frac{Q^2 S}{D_{50}^{0.5} B D} \tag{18}$$

Applying $\Delta Qs/Qs$=0 to Equation 18 and solving for ΔB yields

$$\Delta B = B\{2\frac{\Delta Q}{Q} - \frac{1}{2}\frac{\Delta D_{50}}{D_{50}} + \frac{\Delta S}{S} - \frac{\Delta D}{D}\} \quad (19)$$

Equation 19 is similar in form to Equation 17, while differences lie in the value of the coefficients.

Extremal Energy Slope

The energy slope (S) is expressed from the Darcy-Weisbach equation, Equation 10, as

$$S = \frac{Q^2 f_r}{8gD^3B^2} \quad (20)$$

Applying $\Delta S/S$=0 and solving for ΔB yields

$$\Delta B = B\{\frac{\Delta Q}{Q} - \frac{3}{2}\frac{\Delta D}{D} + \frac{1}{2}\frac{\Delta f_r}{f_r}\} \quad (21)$$

If Manning's resistance equation is used for expressing the slope, the equation for ΔB becomes

$$\Delta B = B\{\frac{\Delta Q}{Q} - \frac{5}{3}\frac{\Delta D}{D} + \frac{\Delta n}{n}\} \quad (22)$$

where n is the Manning's roughness coefficient.

Extremal Stream Power

The stream power ($P= \gamma\ Q\ S$), using the Darcy-Weisbach equation to express the energy slope, is given as

$$P = \gamma QS = \frac{\gamma}{8g}\frac{Q^3 f_r}{D^3B^2} \quad (23)$$

Applying $\Delta P/P$=0 and solving for ΔB yields

$$\Delta B = B\{\frac{3}{2}\frac{\Delta Q}{Q} - \frac{3}{2}\frac{\Delta D}{D} + \frac{1}{2}\frac{\Delta f_r}{f_r}\} \quad (24)$$

When using Manning's resistance equation, the resulting equation for ΔB is

$$\Delta B = B\{\frac{3}{2}\frac{\Delta Q}{Q} - \frac{5}{3}\frac{\Delta D}{D} + \frac{\Delta n}{n}\} \quad (25)$$

Extremal Unit Stream Power

The unit stream power (VS) can be expressed as:

$$VS = \frac{\sqrt{8gDS}}{\sqrt{f_r}}\frac{Q^2 f_r}{8gD^3B^2} = \frac{Q^2\sqrt{f_r S}}{D^{5/2}B^2\sqrt{8}} \quad (26)$$

Applying $\Delta(VS)/(VS)$=0 and solving for ΔB yields

$$\Delta B = B\{\frac{\Delta Q}{Q} + \frac{1}{4}\frac{\Delta f_r}{f_r} + \frac{1}{4}\frac{\Delta S}{S} - \frac{5}{4}\frac{\Delta D}{D}\} \quad (27)$$

Extremal Froude number

The Froude number (F) can be expressed as

$$F = \frac{V}{\sqrt{gD}} = \frac{Q}{BD^{3/2}\sqrt{g}} \tag{28}$$

Applying $\Delta F/F$=0 and solving for ΔB yields

$$\Delta B = B\{\frac{\Delta Q}{Q} - \frac{3}{2}\frac{\Delta D}{D}\} \tag{29}$$

Assuming constant depth and slope (ΔD = 0 and ΔS = 0), Equations 14, 17, 19, 21, 22, 24, 25, 27, and 29 indicate that channel width is directly proportional to the discharge, i.e. channel-width increases when the discharge increases and vice versa in accord with regime equations and field data. Assuming constant discharge and slope, the same equations indicate that channel width is inversely proportional to changes in channel depth, i.e. channel depth decreases (channel fill) when channel width increases, and when channel depth increases (channel scour), channel width decreases. This is in agreement with the observations that river channels tend to widen when aggrading and to become narrow when degrading. In addition, according to Equations 21, 22, 24, 25, and 27, channel width is directly proportional to channel roughness, i.e. increasing channel roughness causes the channel to become wider and vice versa.

On the Regime Theory

The variational approach suggested herein is used to express the response of regime channels to changes in flow discharge. Applying the variational approach to the regime equation for the channel top width, Equation 2, yields

$$\Delta B = abQ^{b-1}\Delta Q \tag{30}$$

Dividing by Equation 30 by Equation 2 yields

$$\frac{\Delta B}{B} = b\frac{\Delta Q}{Q} \tag{31}$$

Similarly, applying the same procedure to the rest of the regime equations, Equations 3-5, yields

$$\frac{\Delta D}{D} = f\frac{\Delta Q}{Q}, \qquad \frac{\Delta V}{V} = m\frac{\Delta Q}{Q}, \qquad \frac{\Delta S}{S} = j\frac{\Delta Q}{Q} \tag{32}$$

In this form, the regime equations give the relative change of channel width, depth, slope, and velocity which allows various values of these variables for a single discharge, depending on whether the discharge is increasing or decreasing. However, under the regime equations, the discharge is the only controlling variable and no interrelation between the width, depth and slope is expressed.

Connection between Extremal Methods and Regime Theory

By substituting Equations 31 and 32 based on the regime equations into any of the preceding extremal equations, the connection between the two methods and a more general form of the hydraulic exponents are obtained. For example, substituting Equations 31 and 32 into Equation 14 for the extremal friction factor yields

$$b = 1 - 1.5f - 0.5j \tag{33}$$

Equation 33 can be considered as a modified general form of the regime theory equations. It shows the interrelation between the dependent variables such as channel width, depth, and slope. In this form the width exponent "*b*" is no longer a constant but depends on the depth and slope exponents. This form also makes it very convenient to compare all the methods as was done in Hafez (1999). For example, if the depth exponent "*f*" is taken as 1/3 and slope exponent as j=0 (constant slope), then the width exponent "*b*" becomes 0.5 according to Equation 33. This value agrees with the measured field value of b=0.5 in regime canals. The same value of b as 0.5 results from Equation 29 for the Extremal Froude number concept. Therefore, the regime theory power equations are special cases of the extremal methods.

APPLICATION TO FIELD DATA

In the following sections several applications of the various extremal equations developed earlier are implemented for field cases. The objectives are to test the validity and quality that these equations provide and show how these equations can provide valuable information that can be useful for understanding the dynamic adjustments of river channels. No intent is given here for selecting one method as the best. This stems from the finding that different extremal criteria exist in the dynamic adjustment process depending on the state of the river channel regime, such as being at lower or higher flow regime. Environmental changes manifested in the form of severe floods that caused substantial channel changes, and in some cases bridge collapses, are investigated via predicting channel width changes. In addition, man-made environmental interference through dam construction induced channel width changes are investigated. The environmental negative side effects of channel width reduction are clear. Finally artificial cutoff and channel slope changes are studied.

Santa Margarita River

Chang (1988) reports general scour at the Basilone Road bridge on the Santa Margarita River for which he made a numerical simulation. A major flood (nearly the 50-year flood) of about 1359 m^3/s occurred in the winter of 1978. At this discharge the adjacent flood plain had a width of about 305 m which was about five times the width of the bridge opening. At low flow the channel had a width of 30.5 m, and no constriction of flow occurred through the bridge opening. Inspection of scour at the bridge footings, which reached 3 m below the river bed, revealed broken reinforced concrete pile. A 12.2 m lower section was never found, which might have been washed away by the flood. Chang simulated numerically the flood event and predicted a scour depth at the bridge site to be about 4.7 m during the rising limb of the flood hydrograph. After the falling period of the flood, channel width decreased and flow constriction effects diminished, with the result that the channel bed at the bridge was filled by sediment to about 3.3 m depth.

In order to estimate changes in the channel bed during the passage of the flood, it is assumed that changes in the channel flow depth are equal to changes in channel bed levels. It is further assumed that slope effects could be neglected. With no change in channel width observed at the bridge section, Equation 14 for the extremal friction factor simplifies to

$$\Delta D = \frac{2}{3} D \frac{\Delta Q}{Q} = \frac{2}{3} D \frac{Q_2 - Q_1}{\left(\frac{Q_2 + Q_1}{2}\right)} \tag{34}$$

Based on the data reported by Chang, Q_2 is taken as 1359 m^3/s, Q_1 as zero, and D as 3m, which nearly equals the bankfull depth. Substitution of these data yields ΔD as 4.1m (positive ΔD indicates scour of the bed, i.e. $D2$>D1) which is close to the value of 4.7m according to Chang's simulation runs. For the falling limb of the hydrograph the input data are Q_1 = 1359 m^3/s, Q_2 = 113m^3/s, and D = 3m which are substituted into Equation 52. This yields ΔD = - 3.1m (deposition or bed rising) while Chang's data indicates 3.3m of bed rising.

Prediction of the average width along the channel away from the bridge section during the flood could be made by assuming that there is no significant change in the bed along the channel as seen from Chang's data and assuming minor slope effects. This results in the following simplified equation based on the extremal friction factor

$$\Delta B = B\frac{\Delta Q}{Q} = B\frac{(Q_2 - Q_1)}{(\frac{Q_2 + Q_1}{2})} \tag{35}$$

Substituting Q_2 = 1359m^3/s, and assumed pre-flood bankfull values of Q_1 = 142 m^3/s and B = 30.5m in Equation 35 yields ΔB = 262.1m, while the field data value is 274.3 m.

Equation 35 can also be used to estimate extreme flood discharges (the 50 or 100 year flood in this example) based only on the observation that the channel width changed from 30.5 m before the flood to 305 m after the flood. Equation 35 can be transformed using upstream difference to the form

$$\Delta Q = Q\frac{\Delta B}{B} = Q\frac{(B_2 - B_1)}{B_1} \tag{36}$$

Substituting B_1 = 30.5m, B_2 = 305m, and Q = 142m^3/s (an assumed bankfull discharge) yields ΔQ = 1274m^3/s. Therefore the predicted flood discharge = (1274+142) m^3/s= 1416 m^3/s which is in excellent agreement with the observed value of 1359m^3/s.

From the foregoing example it is concluded that even with the simplified form of the extremal equations that predict the channel flood width, and scour and deposition depths, excellent agreement is reached with both field data and predictions of rigorous mathematical models. In addition, the proposed equations can predict extreme flood discharges and channel widths, an option which is not easily predicted by the complicated mathematical models.

San Dieguito River

The San Dieguito River at Santa Fe, California had two floods (Chang 1988). The first occurred in March 1978 (peak flow = 125m^3/s), and the second in February 1980 (peak flow = 623 m^3/s). The bridge on Via de Santa Fe road was damaged on February 21, 1980 as a result of channel bed scour and high velocities. Measured field data indicate that at the bridge section during the first flood, the channel bed had scoured by about 4m (bed level changed from about 7.6m to 3.7m) and the width remained constant. Applying Equation 34 for the extremal friction factor while taking ΔB = 0, Q_1 = 0.0, Q_2 = 125m^3/s, and D as 3 m (assumed bankfull depth) yields ΔD = 4.1m (scour) while the field data value was 4m. The water depth is =D+ΔD = 3+4.1= 7.1m. After the second flood the scour hole at the bridge section was filled (the bed elevation rose from about 3.7m to 8.2m). However, channel widening occurred at the right bank where the channel width went from 106.7m to 131.1m, which caused the bridge to collapse near the right bank. With Q_1 = 125m^3/s, Q_2 = 623m^3/s, and D = 7.1m, Equation 34 yields ΔD = - 6.3m (bed rising) compared to observed field value of -4.6m. It should be noted that width changes were not incorporated in this calculation. When width changes are incorporated the resulting ΔD becomes – 5.5 m, which is a much better improvement. If extremal sediment efficiency is used, ΔD is predicted to be - 4.1. The south bank at the bridge section was constrained by hills, which inhibited prediction of width changes as it is assumed that the banks are free to move. However, with knowledge of bank resistance, width prediction could be improved using the extremal equations.

San Diego River - Breach Morphology

A small flood cut a breach through the sand ridge at a bridge crossing on the San Diego River during 1978 (Chang, 1982). Changes of the breach morphology were characterized by initial gully formation through the sand ridge and its gradual widening and decrease in scour depth. The measured flood hydrograph shows that initially the discharge was about $18 m^3/s$ which rose quickly to $71 m^3/s$ in about 11.5 hrs. With $Q_1 = 18 m^3/s$, $Q_2 = 71 m^3/s$, and $D = 1.5m$ (assumed bankfull depth at the ridge), Equation 34 predicts the scour depth after 11.5 hrs to be 2.7m compared with Chang's FLUVIAL-12 model prediction of 2.6m. With $Q_1 = 18\ m^3/s$, $Q_2 = 71 m^3/s$, and $B_1 = 30.5$ m the corresponding width at 11.5 hrs using Equation 35 is 66.1 m while Chang's predicted a width of about 64 m. The final measured breach dimensions are depth of 2.1 m and width of about 82.3 m while the predicted values are depth of 2.6 m and width of 66.1 m. Formation of two large borrow pits upstream and downstream of the sand ridge due to sand mining before the flood makes it difficult to accurately predict the breach final dimensions using the equations developed in this paper. Indeed, the phenomena then have higher dimensions that can be difficult to analyze by equations based on a one-dimensional assumption.

San Elijo Lagoon

The San Elijo Lagoon on the southern California coast was flushed on February 4, 1975 (Chang, 1988) in order to drain storm water stored in the lagoon. Chang and Hill (1977) made a simulation study of the stream-delta system. The delta formed at the channel mouth was very wide and fan shaped during the rising tide period. The rising tide was associated with a decreasing velocity in the channel and an increasing width at the delta section. Conversely, the lowering tide was associated with an increasing velocity in the channel and a decreasing width at the delta. Chang (1988) reported a maximum delta width of 211.2 m and a minimum width of 29.9m based on minimization of the total stream power. The discharge was initially 4 m^3/s and rose to 5.7 m^3/s in about 7 hrs. Applying Equation 35 with $Q_1 = 4\ m^3/s$, $Q_2 = 5.7\ m^3/s$, and $B = 152.4$ m results in $\Delta B = 53.8$ m and hence $B_2 = 206.2$ m which compares well with Chang's value of 211.2 m in the rising period. During the falling tide, the increase in velocity in the channel led to intensive sediment movement, which suggests applying the width equation according to the extremal sediment efficiency concept for which the simplified equation is

$$\Delta B = 2B\frac{\Delta Q}{Q} \tag{37}$$

With $Q_1 = 5.7\ m^3/s$, $Q_2 = 4\ m^3/s$ and $B = 206.2$ m, Equation 37 yields $B_2 = 28.9$ m which compares well with Chang's value of 29.9 m. If a value of $B_1 = 213.4$ m were used, then B_2 would be 30.5 m. If Equation 35 were used instead then B_2 would be 138.1 m. It should be noted that the reported measured width was 42.7 m during the low tide, and then finally dropped to 15.2 m with an average of the two values of 29 m. This value could be considered as the measured width value for comparison with the predictions made here while noting the difficulty of defining the width at the delta mouth during highly transient phenomena such as this.

Change in Width Resulting from Dam Construction

In this section, three selected cases are discussed to illustrate reduction of channel width due to dam construction and associated reduction of released discharges. Assuming for the three cases that the variation of channel width is only due to variations in the discharge, the extremal methods of friction factor, Froude number, and energy slope all are expressed as $\Delta B/B=\Delta Q/Q$ which is used in Table 1 for width change predictions. Table 1 shows width predictions according to the above equation, those predicted by Milhous (1988) using Equation 6 and the observed field values assembled by Williams and Wolman (1984).

It is clear from Table 3 that the current extremal methods give superior predictions to those of Milhous (1988) and agree very well with field data.

The Nile River, Egypt

This case is about width reduction in the Nile River, Egypt after the construction of Aswan High Dam. Field data, (Nile Research Institute, 1992) indicate that the maximum flow below the dam in the fourth reach (Km 545 to Km 953 below High Aswan Dam) decreased from 7000 m^3/s to 2000 m^3/s. Accordingly, channel width decreased on the average from 1000 m to about 540 m. If the depth is assumed to vary as 1/3 of the discharge and slope is assumed constant, the extremal friction factor yields $\Delta B=0.5\,B\,\Delta Q/Q$. With Q_2=2000 m^3/s, Q_1=7000 m^3/s, and B=1000 m, the preceding equation results in a predicted width of 444 m. If it is assumed that this width prediction is accurate, then the channel reach has reached 82 percent of the equilibrium width. On the other hand if it is assumed that slope varies with –1/6 power of the discharge, the extremal friction factor yields $\Delta B = 0.42B\,\Delta Q/Q$. Application of this equation using the same data yields a width of 534 m that is very close to the observed width of 540 m. In this case the channel has reached 99 percent of equilibrium conditions.

Response of the Mississippi River to Cutoffs

Artificial cutoffs are often used to improve navigation and reduce flood-level through straightening of curved reaches of river channels. A very intensive cutoff program was implemented on the lower Mississippi River which revealed that a complete knowledge of cutoffs is necessary before any future plans are made. The reach on the lower Mississippi River between the Arkansas River junction and

Table 1. Change in Width Resulting from Dam Construction

River	Average Annual Peak Discharge (m^3/s)		Channel Width Below Dam (m)		Predicted Width Current Approach (m)	Predicted Width Milhous 1988 (m)
	Pre-dam	Post-dam	Pre-dam	Post-dam		
Jemez River NM, USA	160	39	213	46	56	99
Arkansas River CO, USA	560	190	152	45	51	85
Wolf Creek OK, USA	240	35	223	31	33	79

Greenville had a series of cutoffs constructed from 1933 to 1937. The sinuosity of this reach was reduced from about 3 to 1.4 while bankfull discharge of 31000 m^3/s remained essentially constant. The river responded to cutoffs (Chang, 1988) by forming a wide and even braided channel. This called for an extensive levee system to be constructed in order to maintain channel alignment. The measured bankfull average width in 1933 was 1310 m which reached about 2000 m in 1975. Chang (1988) predicted the 1975 width to be 2800 m and attributed the difference to the levee system causing inhibition of width development. The equation for extremal energy slope or stream power is used herein as the phenomena considered here are related to channel slope changes. The slope can be included in both equations. This is possible by assuming, according to Equation 10, that the slope is proportional to the friction factor. Then $\Delta f_r/f_r$ can be replaced by $\Delta S/S$. With ΔQ=0 (constant bankfull discharge) and assuming ΔD=0, both of Equations 21 and 24 become

$$\Delta B = \frac{B}{2}\frac{\Delta S}{S} \tag{38}$$

Further it is assumed that the sinuosity = (valley slope/channel slope), and that the valley slope remained constant. This result is that the sinuosity is inversely proportional to channel slope. Therefore, using upstream difference, Equation 38 after substitution of data becomes

$$\Delta B = \frac{1310}{2}\frac{\left(\frac{1}{1.4}-\frac{1}{3}\right)}{\frac{1}{3}} \cong 749m \tag{39}$$

thus the final predicted channel width is B = (1310+749) = 2059 m, which compares very well with the observed value of 2000 m. If the levee system had not been implemented, this value is a good prediction of the final equilibrium width. This example is different in nature from the preceding examples and shows the versatility and power of the developed equations from extremal methods.

Chippewa River

The Chippewa River of Wisconsin was studied by Schumm and Beathard (1976) and Chang (1988). This is a case of channel downstream change from sinuous and narrow upper upstream reach to a wider and braided lower downstream reach. The downstream reach ends at the confluence of the Chippewa River with the Mississippi River. The development of the new downstream channel following abandonment of the Old Buffalo Slough was attributed by Schumm and Beathard (1976) to bank erosion, which resulted in intensive sediment transport. This nearly straight reach has a bankfull width of 305 m, a medium grain size of 0.53 mm, a channel gradient of 0.000333 and a valley slope of 0.000347. The upper sinuous channel has a bankfull width of 195 m, a medium diameter of 1.93 mm (according to Chang (1988), Fig. 11.11), a channel slope of 0.000278 and a valley slope of 0.000403. The bankfull discharge for the river is given as 1645 m^3/s. Chang (1988) using charts based on the power approach predicted the lower channel width as 380 m and the upper channel width as 210 m. It will be assumed that this case of downstream change can be transformed to at-a-station change. This means that the lower channel reach can be considered to change from a sinuous and narrow reach to a wider and straight reach. Past values for the variables for the lower reach are assumed equal to the upstream sinuous reach.

Because of the intensive sediment transport in the lower reach, it will be assumed that the extremal sediment transport (using Yang's simplified transport equation) is appropriate. Further the discharge and depth are assumed constants as implied by Chang. Under these conditions, Equation 19 simplifies to

$$\frac{\Delta B}{B} = \frac{\Delta S}{S} - \frac{1}{2}\frac{D_{50}}{D_{50}} \tag{40}$$

Using the above given data for valley slopes, gives

$$\frac{\Delta S}{S} = \frac{(S_2 - S_1)}{\frac{(S_2 + S_1)}{2}} = \frac{(0.00347 - 0.000403)}{\frac{(0.000347 + 0.000403)}{2}} = -0.149 \tag{41}$$

Similarly the term $\Delta S/S$ is equal to 0.18 when using channel slopes.

Using the above given data for the median grain size term yields $\Delta D_{50}/D_{50}$ = -1.138. Equation 40 predicts $\Delta B/B = 0.42$ using valley slopes and 0.744 using channel slopes. The term $\Delta B/B$ is expressed as

$$\frac{\Delta B}{B} = \frac{(B_2 - B_1)}{\frac{(B_2 + B_1)}{2}} \tag{42}$$

Taking B_1 as 195 m, Equation 42 predicts new channel widths of 299 m and 429 m using valley and channel slopes respectively. The smaller value of channel width (299 m) corresponds to using valley slopes and is consistent with having small variations in the variables. Thus the lower reach new width is predicted as 299 m compared with a field value of 305 m which constitutes an excellent agreement. This example shows the effects of grain size and slope on width changes.

From the foregoing applications, it is seen that the predictions of channel response using the response theory equations work very well for both short (transient) and long term channel adjustments.

Summary of Predictions

Figure 1 shows plot of predicted and measured channel widths which nearly coincide with the line of perfect agreement. Tables 2 and 3 show a summary of the width, and scour and deposition depth predictions implemented in the preceding sections along with field data for comparison purposes.

SUMMARY AND CONCLUSION

A response theory is presented for predicting qualitatively and quantitatively the equilibrium width, depth, and slope of alluvial river channels. The theory makes use of the tendency of alluvial channels toward dynamic equilibrium. Various extremal concepts are adopted such as extremal energy dissipation (including its special cases of extremal stream power, unit stream power and energy slope), extremal sediment efficiency, extremal friction factor, and extremal Froude number. Selection is made of the function that describes each concept in terms of the controlling or independent variables (e.g. water discharge) and the adjusted or dependent variables (e.g. width, depth, slope, roughness, and sediment size). An equation that represents the dynamic adjustment results in setting the variation of the function to zero. This is the condition for the function to have an extremal value, either a maximum or minimum.

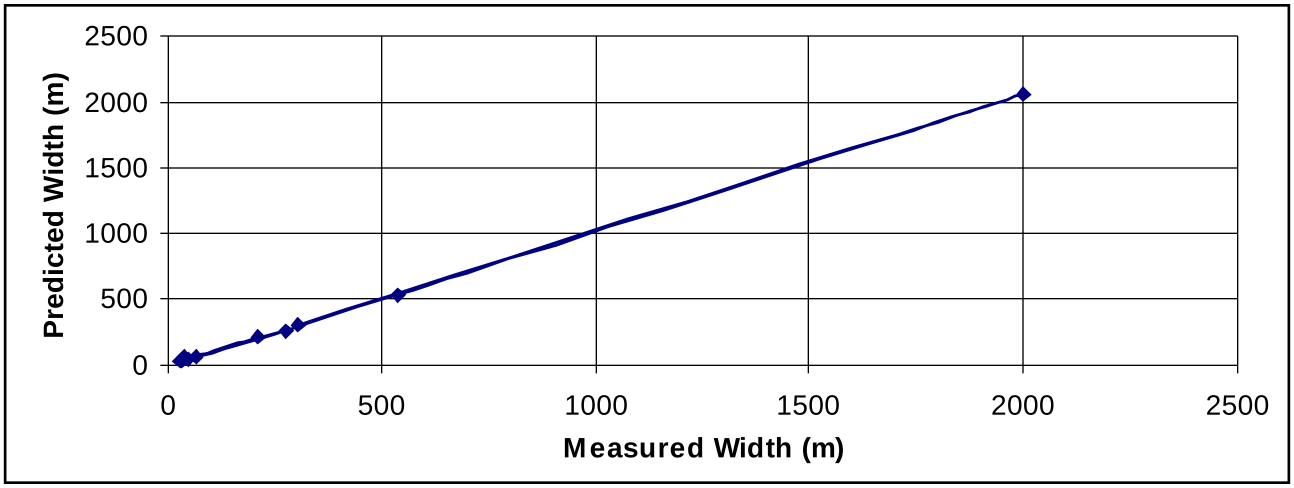

Figure 1. Predicted versus measured width.

Table 1. Measured and Predicted Channel Width

River	Measured Width (m)	Predicted Width (m)	Ratio (predicted/measured)
Santa Margarita River, CA	274	262	0.96
San Diego River, CA	64	66	1.03
San Eligo Lagoon, CA, Falling Tide	211	206	0.98
San Eligo Lagoon, CA, Rising Tide	30	29	0.97
Jemez River, NM	46	56	1.22
Arkansas River, CO	45	51	1.13
Wolf Creek, OK	31	33	1.06
Nile River, Fourth Reach, Egypt	540	534	0.99
Mississippi River, USA	2000	2059	1.03
Chippewa River, WI	305	299	0.98

Table 3. Measured and Predicted Scour and Deposition Depths

River	Measured Depth		Predicted Depth		Ratio
	(m)	(ft)	(m)	(ft)	(predicted/measured)
Santa Margarita River, CA	-3.3	10.7	-3.1	10.30	0.94
San Dieguito River, CA	-4.6	15.0	-6.3	20.7	1.37
San Dieguito River, CA	+4.0	13.0	+4.1	13.30	1.03
San Diego River, CA	+2.6	8.5	+2.7	8.9	1.04

Note: negative sign indicates bed rising and positive sign indicates bed lowering

All extremal method adjustment equations predict channel widening and bed rising during the rising period of the flood hydrograph, while predicting channel narrowing and bed lowering in the falling period of the flood hydrograph. Thus, the interrelationship between width, depth and slope is accounted for correctly. The equations for predicting channel adjustments using the various extremal methods are similar in form as they predict the same direction of adjustment while they differ in predicting the magnitude of the adjustment.

The regime equations can be derived from the extremal equations developed herein provided that the discharge is the only controlling variable. The equations developed from extremal concepts predict successfully the hydraulic exponents in the regime type equations. Extremal friction factor and Froude number gives a value of the hydraulic exponent b as 0.5 which coincides with the data of regime canals in India and Pakistan.

Applications of the extremal methods to field data show the success in accurately predicting changes or adjustments in the width and depth of natural channels. The ratio of predicted width values to measured ones ranged between 0.96 to 1.06 for 8 out of 10 cases. For the other two cases the ratios are 1.13 and 1.22 which still constitutes a good agreement to the prediction of a complex phenomena. For prediction of depth of scour or deposition, the ratio ranged between 0.94 to 1.04 for 3 out of 4 cases.

The prediction of adjustments in width and depth of alluvial channels made herein is considered excellent. Considering the complexity of the channel adjustment phenomena, the current approach is very promising due to its strong theoretical background, simplicity in application, and minimum data requirements. Predictions are possible using the current approach of magnitudes of flood peaks and channel forming discharges.

The writer advocates the use of all the equations here to predict both maximum and minimum expected values of the width, depth, and slope. This provides an envelope of expected future values and gives more reliability than predicting only a single value. Indeed, channel adjustment is a complex phenomena that cannot be predicted exactly. The extreme variability of estimating sediment transport in natural channels is just one example of the inherent difficulty in analysis of channel processes.

The adjustment equations developed herein can be used in sediment models of scour and deposition by supplying a relation between adjustments in the width and depth of alluvial channels.

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