JOURNAL OF
ENVIRONMENTAL HYDROLOGY

The Electronic Journal of the International Association for Environmental Hydrology
On the World Wide Web at http://www.hydroweb.com

VOLUME 10 2002

ON THE DYNAMIC ADJUSTMENTS OF STREAM CHANNELS

Youssef I. Hafez
Nile Research Institute
Water Research Center, Delta Barrage
El Kanater, Greater Cairo, Egypt

The hydraulic exponents which appear in the regime theory equations are predicted using different methods such as an empirical function, extremal energy slope, or extremal stream power. These hydraulic exponents represent the dynamic adjustments or response of river channels to changes in their regime. Based on experimental and theoretical data from examples of channel adjustments, an empirical function is developed which, when varied, yields the hydraulic exponents. Extremal energy slope in the form of minimum or constant energy slope, and extremal stream power in the form of minimum or constant stream power, are both derived by setting the variation of their corresponding function to zero, which results in a relation between the hydraulic exponents. Four examples for which data exist regarding the values of the hydraulic exponents are used to validate the empirical function and extremal methods. The first example is the channel response to changes in flow over a sand bed between rigid walls at constant slope. This example resembles river reaches where the banks are firm (either stiff clay or protected by riprap) or at gauging station cross-sections having relatively stable banks. In this case the flow has one degree of freedom to adjust by changing its roughness. Parallel to this case is the case in which the channel cross section, at constant slope, has a constrained width in the form of a relation between the width and depth, which accounts implicitly for the degree of bank resistance. The third example having two degrees of freedom is the response of the river cross section to change in discharge at constant slope by adjusting its width and depth. This resembles river sections where the banks are loose and free to move. The fourth example having three degrees of freedom is when the channel adjusts its slope (longitudinal profile), depth and width to accommodate for the downstream increase in discharge. This case resembles rivers in humid regions where flow increases in the downstream direction due to incoming tributary flows. The hydraulic exponents in each case are presented and compared to theoretical and field values along with discussion of the inherited mechanisms.

INTRODUCTION

Environmental and man-made interferences often exert changes in the input of water and sediment discharges brought down to river channels. This causes stream channels to undergo continuous change or dynamic adjustment in their regime due to changes of input water and sediment loads. Although channel adjustment is a complex process, it can be described in terms of mean values of the geometrical and hydraulic variables. Therefore, hydraulic geometry of river channels can be described in terms of mean or average top width, average depth, average cross sectional velocity and average longitudinal slope. Upon undergoing a change process, rivers have certain degrees of freedom to adjust. Chang (1988) illustrated that straight river channels have in general four degrees of freedom: depth, width, channel-slope and bank slope. Depth is determined according to a stage-discharge or flow resistance relationship. Channel slope can be computed using a sediment transport equation. For sand and gravel banks, bank slope is determined by the angle of repose, whereas for silt and clayey banks, bank slope is determined by the silt-clay contents of the banks. Up to this point, commonly used hydraulic principles have been fully utilized, while there is no apparent physical law to determine the channel liberty to adjust its width. A width predictor formula needs to be prescribed in order to understand the behavior of river channels.

To determine a criterion for width computation, two commonly methods have been reported, namely; regime methods and extremal methods. In regime methods, width is expressed in the form of a power function of bankfull discharge. Similar expressions are used in regime methods to determine the depth, velocity and slope as power function of the discharge. The general form of these relations at a channel cross section (at-a-station hydraulic geometry) is

$$B = aQ^b \tag{1}$$

$$D = cQ^f \tag{2}$$

$$V = kQ^m \tag{3}$$

$$S = iQ^z \tag{4}$$

in which B is the channel top width, Q is the water discharge (usually bankfull discharge), D is the average depth, V is the average velocity and S is the channel slope. The constants a, b, c, f, k, m, i and z are empirical constants. The exponents b, f, m and z are termed the hydraulic exponents. These hydraulic exponents represent the dynamic adjustments or response of river channels to changes in their regime. Equations 1, 2 and 4 describe the geometrical parameters of the river channel as a function of hydraulic variables such as the discharge. Hence the name hydraulic geometry is used in this context. Because Q=VBD, the following identities result in

$$b + f + m = 1 \tag{5}$$

$$ack = 1 \tag{6}$$

Identity (5) will be referred to as the unity identity.

Leopold and Maddock (1953) used 20 river cross-sections in the Great Plains and the Southwest USA to determine the hydraulic exponents. They reported values of b from 0.03 to 0.59 with a mean of 0.26, values of f from 0.03 to 0.63 with a mean of 0.4 and values of m from 0.07 to 0.55 with a mean of 0.34. Lacey (1930, 1958) using data from canals assumed in regime in India and Pakistan, determined b as 0.5 and z as –1/6. Blench (1952, 1970), following Lacey's approach, determined b as 0.5, f as 1/3, m as 1/6 and z as –1/6. Simons and Albertson (1960) using India and Pakistan canals and others in Colorado, Wyoming and Nebraska in USA, determined b as 0.5 and f as 0.36. In the downstream direction where discharge increases due to tributary flows, the same form of the regime equations is used while values of the coefficients and exponents differ. Leopold and Maddock (1953) using field data, determined b as 0.5, f as 0.4 and m as 0.1.

Extremal methods on the other hand adopt the concept that river channels adjust to equilibrium after being naturally or artificially disturbed by having an extremal condition occurring (either maximum or minimum). Mackin (1948) stated that the river channel adjusts itself in order to accommodate the transport of debris.

Langbein (1964) presented the theory of minimum variance to theoretically derive the hydraulic exponents. The variance was defined as the sum of the square of the hydraulic exponents to be set to a minimum consistent with local restrictions. Langbein stated that stream channels have a mean form (hydraulic geometry) that must fulfil the necessary hydraulic laws, but in addition, a river channel tends toward equal distribution among velocity, depth, width and slope in accommodating change in stream power. This means that $b = f = m = 1/3$. Departure of these values in natural channels was attributed to the imposed constraints on river channels. He presented three examples to show the applicability of the theory of minimum variance.

The first example by Langbein (1964) is that of the response to changes in flow over a sand bed between the fixed walls of a circulating flume at constant slope. The flow has one degree of freedom; the liberty to adjust its roughness. The minimum variance theory yields $f = 1/2$ and $m = 1/2$ in this case. This means that in a fixed-width channel, at constant slope, a change in discharge tends to be accommodated equally by changes in velocity and depth. Hence, the friction factor in the Darcy-Weisbach formula would decrease as the square root of the discharge. This finding was supported by experiments of Simons et al. (1961) where the friction factor for constant width and slope channel varied with the –0.48 power of the discharge. Field data of the River Nile in 1988 after the construction of Aswan High Dam, (Nile Research Institute 1992), indicated that $f = 0.46$ and $m = 0.5$ for the five gauging stations which support the above finding. The banks at the gauging stations are fairly stable and hence width can be considered constant. In addition, slope was found to vary little with discharge.

The second example by Langbein (1964) is that of the accommodation of a river channel at a given cross section to changing discharge. Herein, water-surface slope is assumed constant while velocity, depth and hydraulic resistance are the dependent factors. Langbein assumed that for the stable cross section the forces applied to the bed and banks are distributed uniformly. The cosine equation derived by Nizery and Braudeau (1955) as a limiting condition on the width gives $b=0.55\,f$. The minimum variance theory then yields $m = 0.35$, $f =0.42$ and $b = 0.23$. Thus, the friction factor decreases as the 0.28 power of discharge while the Manning roughness would decrease only as the 0.07 of the discharge. Langbein observed the decrease in the friction factor with discharge in this case is less at a river section where the width increases with discharge, than in the preceding example of a fixed width. If the width is not constrained or completely free to adjust, the friction factor would decrease less than its value in the last two cases.

Williams (1978) using data for 74 stations with firm banks examined the hypothesis that $b = 0.55\,f$ and found that this value is approximated or exceeded in only three cases. He obtained an average value of $b = 0.19f$. However, plots of b versus f indicate that $b = 0.12\text{-}0.06f$ and that b tends to decrease with increase in f. For two thirds of the 74 cases, the average value of b was 0.08. Williams, using data of these 74 stations, reported average field values of m = 0.42, f = 0.5 and b = 0.08 that differ significantly from the values by Langbein.

The third example by Langbein (1964) is that of a river reach in a humid region with the liberty to adjust its profile (slope), velocities, depths and widths to accommodate the downstream increase in discharge. There are three degrees of freedom, namely; width, depth and slope. By assuming a mean profile between the profile at constant stream power per unit length, γQS, and the profile at constant unit stream power per unit area, $\gamma Q\ S/B$, a mean profile was assumed in which $z = b/2$-1. Further it was assumed that the Manning resistance coefficient n varies as –0.22 power with the discharge. After applying the minimum variance theory, it was found that b = 0.53, f = 0.37, m = 0.1 and z = -0.73. These predictions were supported with field data by Leopold and Maddock (1953) who reported that b = 0.5, f = 0.4 and m = 0.1.

Yang (1987) stated the problem of the selection of the correct combinations of the variables used in the variance minimization and the fact that different combinations lead to different answers. However, Langbein's concept of minimization inspired research into the application of minimization or maximization hypotheses to explain hydraulic geometry.

Yang (1976) introduced the concept of minimum unit stream power, VS. He stated "an alluvial channel with subcritical flow in the lower flow regime tends to adjust its velocity, depth, slope and channel roughness in such a manner that given water discharge and sediment concentrations can be transported with the minimum amount of unit stream power under given geologic and climatic constraints". Later, Yang and Song (1979) introduced the theory of minimum rate of energy dissipation and derived the minimum stream power, γQS, as a special case. The theory was applied by Yang et al. (1981) to the study of channel geometry by theoretically deriving the hydraulic exponents as $b = f = 9/22$, $m = 4/22$ and $z = -2/11$. They stated that channel depth can readily be adjusted in accordance with the theory, while width adjustment may also depend on constraints other than discharge and sediment load.

Hafez (2000) introduced the response theory for predicting adjustments in channel width, depth, velocity and depth. The theory makes use of the tendency of alluvial channels toward dynamic equilibrium. Various extremal concepts are adopted such as extremal energy dissipation (including its versions of total stream power and unit stream power), extremal sediment efficiency, extremal friction factor and extremal Froude number. Selection is made of the function that describes each concept in terms of the controlling and dependent variables. An equation that represents the dynamic adjustment results by setting the variation of the function to zero. This is the condition for the function to have an extremal value, either a maximum or minimum. Derivation of the procedure is introduced here for the sake of completeness only for the relevant cases of extremal energy slope and stream power while the details of the complete procedure is found in Hafez (2000).

It should be noted that during the adjustment process, there is an interrelationship among the width, depth, slope and bank slope of the river channel. Therefore, rather than solving for each variable according to its corresponding physical law separately as implied by Chang's (1988a) approach simultaneous solution of all the applicable physical laws is suggested here .

The foregoing analysis of the role of each individual physical law should be understood as a way of demonstrating the major role and relative effect played by each variable. For example, width is affected by the stage-discharge. If the stage or depth changes (increasing or decreasing), this changes the boundary shear stress. This in turn changes sediment transport and its associated pattern of scour and deposition. Clearly, scour and deposition along channel perimeter controls width variation. The analysis used in this paper gives only additional information for the extra degree of freedom. Therefore, additional information is assumed or given according to empirical or field data to compensate for the other degrees of freedom.

For channel cross sections under equilibrium, supplying information about channel roughness is equivalent to using a stage-discharge formula. This is to be done by supplying variation of the friction factor with discharge. Supplying a width-depth relation in the form of relation between b and f or simply specifying b is similar to specifying bank resistance or bank slope criteria. Applying the unity identity ($b+f+m=1$) is similar to specifying the continuity principle. Using these criteria compensates for three degrees of freedom while supplying an additional relation (e.g. from the extremal hypothesis) that compensates for the extra degree of freedom. For equilibrium of a channel reach, an additional relation between slope and any of the other variables is needed. In all analyses in this paper, all the above criteria which are assumed to represent physical laws are specified first and then the extremal methods or the empirical function is applied to determine the extra degree of freedom.

THE RESPONSE THEORY OF RIVER ADJUSTMENTS

The following assumptions (Hafez 2000) are used in this analysis:

1. equilibrium conditions occur toward maximizing or minimizing a function representing the dynamic equilibrium of the river channel;
2. first order variations of the relevant variables; and
3. prismatic,-one-dimensional, rectangular and wide channels.

The equilibrium conditions of straight river channels is a function of channel discharge, roughness, energy slope, width, depth, sediment discharge and bed material size. In functional form, this can be expressed as:

$$\Delta\Psi = Fct\left(Q, f_r S, B, D, Q_s D_s\right) \tag{7}$$

where Ψ is variable that describes the equilibrium conditions of the river, f_r is the friction factor, Q_s is the sediment discharge and D_s is a measure of bed material size. If there is a change in any of the dependent variables in Equation 7, then, according to the extremal concept, in order to restore equilibrium, the corresponding change of Ψ should be zero, i.e.

$$\Delta\Psi = \sum_i \frac{\partial\Psi}{\partial x_i}\Delta x_i = 0 \quad ,or \quad \frac{\Delta\Psi}{Z} = 0 \tag{8}$$

where x_i is any of the dependent variables in Equation 7. Equation 8 is the general equation that corresponds to an extremal condition (either maximum or minimum) of the function describing or representing equilibrium conditions of the river channel. This function can be the energy slope, stream power, the friction factor, the sediment transport or the Froude number. The following explains how Equation 8 is applied in order to obtain the equations predicting the width, depth and slope changes. It should be noted that the prediction equations for channel width are restricted by bank erodibility which limits increase of channel width and by sediment availability which limits width reduction. In a similar manner, bed armoring limits bed scour and sediment availability limits bed rising. The procedure is fully explained in the case of the extremal energy slope. For the case of extremal stream power the procedure is very similar.

Extremal Energy Slope

The energy slope is expressed by the well- known Darcy-Weisbach formula found in standard textbooks of hydraulics as

$$S = \frac{Q^2 f_r}{8gD^3B^2} \tag{9}$$

where f_r is the Darcy-Weisbach friction factor.

Equation 9 in functional form is

$$S = Fct\left(Q, f_r D, B\right) \tag{10}$$

For the slope to be maximum or minimum, the variation of its function is set to zero, i.e. DS=0, which when applied to Equation 10 yields

$$\Delta S \frac{\partial S}{\partial D}\Delta D + \frac{\partial S}{\partial Q}\Delta Q + \frac{\partial S}{\partial B}\Delta B + \frac{\partial S}{\partial f_r}\Delta f_r \tag{11}$$

Applying ΔS/S = 0 yields

$$\frac{\Delta B}{B} - \frac{\Delta Q}{Q} + \frac{3}{2}\frac{\Delta D}{D} - \frac{1}{2}\frac{\Delta f_r}{f_r} = 0 \tag{12}$$

Equation 12 determines the relation between adjustments in the width, depth, roughness due to change in the discharge under the assumption that energy slope is at an extremum.

Extremal Stream Power

The stream power ($P = \gamma QS$), using the Darcy-Weisbach equation to express the energy slope, is

$$P = \gamma QS = \frac{\gamma}{8g}\frac{Q^3 f_r}{D^3B^2} \tag{13}$$

Applying $\Delta P/P = 0$ yields

$$\frac{\Delta B}{B} - \frac{3}{2}\frac{\Delta Q}{Q} + \frac{3}{2}\frac{\Delta D}{D} - \frac{1}{2}\frac{\Delta f_r}{f_r} = 0 \tag{14}$$

except for the coefficient in the discharge term, Equation 14 is similar to Equation 12.

THE EMPIRICAL FUNCTION FOR THE HYDRAULIC GEOMETRY OF RIVER CHANNELS

In this section a function is derived which when varied yields the hydraulic exponents in the hydraulic geometry regime type equations. By inspection, consider the following expression for this function:

$$\Phi = \frac{D^{\frac{1}{4}}}{V}\sqrt{\frac{S}{B}} \tag{15}$$

where Φ is a function that represents equilibrium conditions of the river channel. At equilibrium conditions the variation of Φ is zero ($\Delta\Phi = 0$ or equivalently $\Delta\Phi/\Phi = 0$). Examples follow to illustrate the applicability of this function to changes in river regime.

Stable River Cross Section with Constant Width and Slope

This example shows channel response to changes in flow over a sand bed between rigid walls at constant slope. This case is found in circulating flumes where the output water and sediment are fed back as the input until equilibrium is obtained between the input and output and the bed is stable in character and form (Langbein, 1964). The flow has one degree of freedom to adjust which is through changing its roughness. Roughness in turn controls the depth and velocity to accommodate change in the input discharges of water and sediment.

Langbein (1964) used the minimum variance theory and obtained $f = m = 1/2$. It follows then that the friction factor varies with the –0.5 power of the discharge. In addition this example simulates stable river cross sections such as those where the banks are firm (either stiff clay or protected by riprap) or at gauging stations having relatively stable banks. Average values for the five gauging stations on the Nile River, Egypt, downstream of the High Aswan Dam based on 1988 data indicate that $b = 0.05$, $f = 0.46$ and $m = 0.5$ (the Nile Research Institute, 1992). Field data for stable channels with vertical banks ($b<0.03$)(Williams, 1978) shows that $f = 0.52$ and $m = 0.47$ which is very close to the above theoretical values.

For constant slope and width, the function Φ has only the depth and velocity to vary. For equilibrium conditions, the variation of Φ is zero or $\Delta\Phi/\Phi = 0$, i.e.

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta D}{D} - \frac{\Delta V}{V} = 0 \qquad \Rightarrow \frac{\Delta D}{D} = \frac{\Delta V}{V} \tag{16}$$

It can be shown that (Hafez 2000)

$$\frac{\Delta D}{D} = f\,\frac{\Delta Q}{Q} \qquad ; \frac{\Delta V}{V} = m\frac{\Delta Q}{Q} \qquad ; \frac{\Delta B}{B} = \frac{\Delta Q}{Q} \tag{17}$$

It follows from Equation 17 that Equation 16 gives $f = m$. Using the unity identity and $b = 0$, it follows that $f = m = 1/2$. Accordingly, the friction factor varies as –0.5 power of the discharge. Therefore, the function Φ yields results in agreement with field data of Williams (1978) Nile River 1988 data, flume-data of Simons et al. (1961), and the minimum variance theory.

Stable River Cross Section with Constant Slope and Constrained Width

With slope constant, applying $\Delta\Phi/\Phi = 0$ to Equation 15, while velocity, depth and width vary, yields

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta D}{D} - \frac{\Delta V}{V} - \frac{1}{4}\frac{\Delta B}{B} = 0 \qquad ? \tag{18}$$

In terms of the hydraulic exponents Equation 18 becomes f-m-1/4 b=0. Following Langbein in adopting the cosine equation for stable cross section where $b = 0.55\,f$ and using the unity identity, it follows that f=0.41, $b = 0.23$ and $m = 0.36$. These values are in excellent agreement with those of the minimum variance theory of Langbein (1964) who obtained $f = 0.42$, $b = 0.23$ and $m = 0.35$. Field data of two river cross sections. Leopold and Maddock (1953) show that $f = 0.4$, $b = 0.26$ and $m =$

0.34, which are close with the empirical function results. The friction factor varies as the –0.31 power of the discharge and the Manning's roughness coefficient as –0.09 compared with values of –0.28 and –0.07 by Langbein respectively.

Stable River Cross Section with Constant Slope and Completely Free Width

In this case the width is assumed to be completely free to adjust. There are two degrees of freedom for which an extra assumption is needed. Yang et al. (1981) assumed that $b = f$ in this case. Yang used this assumption and obtained $b = f = 0.41$ and $m = 0.18$. It was proven that these values of the hydraulic exponents are the limiting values of the field data by Barr et al. (1980). However slope varied as –2/11 power of the discharge. Using Equation 18 while adopting $b = f$ and the unity identity yields $b = f = 0.36$ and $m = 0.28$.

Down Stream Hydraulic Geometry of River Reaches

In this case, the discharge is assumed to increase in the downstream direction. There are three degrees of freedom, namely; width, depth and slope. With slope varying, setting the variation of Φ to zero in Equation 15 yields

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta D}{D} - \frac{\Delta V}{V} - \frac{1}{4}\frac{\Delta B}{B} + \frac{1}{4}\frac{\Delta S}{S} = 0 \tag{19}$$

In terms of the hydraulic exponents Equation 19 becomes

$$f - m - \frac{1}{4}b + \frac{1}{4}z = 0 \tag{20}$$

Adopting Langbein's assumption that $z = b/2\text{-}1$ and using the unity identity, Equation 20 becomes

$$2f + \frac{7}{8}b = \frac{5}{4} \tag{21}$$

Because Equation 21 has two unknowns, an extra relation or assumption is needed. Assuming that the friction factor varies as the –0.5 power of the discharge and using the Darcy-Weisbach equation after eliminating m using the unity identity yield

$$3f + 2b = 2.2 \tag{22}$$

Solving equations 21 and 22 simultaneously yields $b = 0.47$ and $f = 0.42$. It follows that $m = 0.11$ and $z = -0.77$. These values compare very well with Langbein's values of $b = 0.53$, $f = 0.37$, $m = 0.1$ and $z = -0.73$ based on the minimum variance theory. Leopold and Maddock (1953) reported from field data that $b = 0.5$, $f = 0.4$ and $m = 0.1$.

Based on these examples, it follows that the suggested function Φ in Equation 15 provides a good simulation of river channel changes for the conditions considered. The advantage of this function is that information about the friction factor is not required, except for the case of downstream hydraulic geometry. However, the function lacks a physical meaning and theoretical background which suggests further research in these aspects. Extremal methods on the other hand have both physical meaning and a strong background. A test of the quality of extremal methods is made in the following sections by predicting the hydraulic exponents in the preceding examples in addition to other cases.

THE EXTREMAL ENERGY SLOPE AND STREAM POWER CONCEPTS

Two extremal methods are selected herein, namely; extremal energy slope and extremal stream power. These methods were selected for the following reasons. The constancy of the slope at a channel cross section in the preceding examples suggests that this slope corresponds to a stationary value that could be either maximum or minimum. For a long-term channel adjustment, a minimum slope is attained for equilibrium conditions to occur. For short-term channel adjustment, a maximum slope is attained. Robbins and Simon (1983) show that after man-induced engineering work at Halls, South Fork ,Forked Deer River, Tennessee, USA, the unit stream power reaches a maximum first in a relatively short term and then eventually decays to a minimum in the long term adjustment process. Therefore, the extremal energy slope criteria is selected for the cases of channel cross sections with constant slope. It is not the intent here to seek whether this slope is a maximum or minimum which, as was just explained, depends on the time scale of channel adjustment.

For a river channel reach where the discharge increases in the downstream direction, slope is no longer constant. The postulate that stream power has a constant value seems appropriate. The constancy of stream power implies a stationary value either maximum or minimum. Leopold and Maddock (1953) and Langbein (1964) implied such possibility. The two concepts are applied to the aforementioned cases. Variation of the friction factor will be assumed given according to experimental and field observations. Indeed, information about roughness or friction is needed for description of river channels as They are often needed when using mathematical models to predict channel changes.

Stable River Cross Section with Constant Width and Slope

Equation 12 for the extremal energy slope can be expressed in terms of the hydraulic exponents after using $\Delta f_r/f_r = p\,\Delta Q/Q$, where p is the exponent that represents the variation of the friction factor as power of the discharge, i.e.

$$b = 1 - \frac{3}{2} f + \frac{1}{2} p \tag{23}$$

As there is one degree of freedom in this case, information is required for the exponent p. It is assumed the friction factor to varies as –0.5 power of the discharge according to laboratory data of Simons et al. (1961), i.e. P = -0.5. For firm banks b = 0 which together with p = -0.5 give f = 1/2 according to Equation 23. It follows from the unity identity that m = 1/2 and from Darcy-Weisbach equation that p = -0.5 which supports the original assumption. The same results were obtained with the empirical function.

Stable River Cross Section with Constant Slope and Constrained Width

Assuming that the friction factor varies with –0.28 power of the discharge and using the assumption that $b = 0.55 f$, Equation 23 gives f = 0.42. With f = 0.42, it follows that b = 0.23 and m = 0.35. These values coincide with those by the minimum variance theory (Langbein, 1964) and agree very well with field data of Leopold and Maddock (1953).

Stable River Cross Section with Constant Slope and Completely Free Width

If the width of the cross section is completely free to adjust, it will be assumed that grain roughness is the only control. In this case the friction factor varies as the –0.2 power of the discharge (Langbein, 1964). With p = -0.2, it follows from Equation 23 that $b = 0.9\text{-}1.5f$. If it is assumed that $b = f$ (equable action assumption) as was done by Yang et al. (1981), it follows that $b = f$ = 0.36 and m = 0.28, which coincides with the values predicted by the empirical function. However, the predictions are slightly different from those of Yang et al. (1981) ($b = f = 0.41$ and $m = 0.18$) due to a varying slope with discharge in Yang's case. Equation 12 with $\Delta f_r/f_r = 0$ and $b = f$ yields $b = f = 0.4$ and $m = 0.2$. The assumption that $\Delta f_r/f_r = 0$ implies that the friction factor is at extremal conditions. The extremal friction factor is at a maximum value that corresponds to a minimum energy slope. Davies and Sutherland (1983a) showed that the conditions at which unit stream power or total stream power is minimized are also those at which friction factor is maximized. For constant discharge, minimum total stream power reduces to minimum energy slope.

Downstream Hydraulic Geometry of River Reaches

The principle of total work adjustment toward a minimum (minimum stream power) is similar to that of the minimum variance, (Langbein, 1964). In other words, there is an equivalency between the principle of equable action and minimum work (stream power). Therefore, extremal stream power is selected herein as the suitable criteria. Equation 14 in terms of the hydraulic exponents can be written as

$$b = \frac{3}{2} - \frac{3}{2} f + \frac{1}{2} p \tag{24}$$

It is assumed that the friction factor varies with slope ($p = z$), as might be inferred from Darcy-Weisbach expression for the friction factor. Using the relation $z = b/2\text{-}1$, it follows that $p = b/2\text{-}1$ which when substituted in Equation 24 yields

$$b = \frac{4}{3}\left\{1 - \frac{3}{2} f\right\} \tag{25}$$

Additional information is needed about either b or f or a relation between the two. If a value of $f = 0.42$ is assumed according to the case of stable river channel cross section, it follows that $b = 0.49$ and $m = 0.09$. With $b = 0.49$ and $z = b/2\text{-}1$, it follows that $z = -0.76$. These values are in excellent agreement with Leopold and Maddock (1953) field data and the minimum variance theory by Langbein (1964). With the same assumptions, the Darcy-Weisbach formula gives $m = 0.21$, $b = 0.37$, $z = -0.82$, while f was assumed as 0.42. The calculated velocity and width exponents are not in good agreement with field data. This example demonstrates the validity of the assumption that the stream power is at extremum (constant stream power) along the river reach. In the following is further application of the extremal energy slope to cases reported by Williams (1978).

Stable Cross Section with Firm Banks and Constant Slope

This case is similar to the stable river cross section with constant slope and constrained width as both have one degree of freedom. The difference is in the criteria for width that reflects different bank resistance for each case. Firm banks are assumed here as those having $b < 0.03$. For a stable cross section with firm banks and constant slope, data of the 74 stations given by Williams (1978) indicate that $b = 0.08$, $f = 0.5$ and $m = 0.42$ which yields $b = 0.16\,f$. Based on the 74 stations, the earlier assumption of Langbein (1964) that $b = 0.55\,f$ was tested. This assumption was found to be not valid based on Williams' data and instead, three possible relations existed. The first is that $b = 0.19\,f$, the second is that $b = 0.12\text{-}0.06\,f$ and the third is that b is simply equal to 0.08. For prediction purposes, it is assumed herein that the friction factor varies as –0.28 power of the discharge as was made in the second example.

For $b = 0.19\,f$ and $p = -0.28$, Equation 23 for the extremal energy slope yields $f = 0.51$. It follows that b = 0.1 and m = 0.39. For b = 0.12-0.06f, Equation 23 gives f = 0.51 and consequently b = 0.09 and m = 0.4. For b = 0.08, Equation 23 gives f = 0.52 and thus m = 0.4. It is clear that extremal energy slope gives excellent agreement with field data for the three assumptions. The cosine-equation based assumption that b = 0.55f is not suitable for cross sections with firm banks and constant slope. For these cross sections it can be assumed that the principle of minimum energy slope is a valid assumption.

Stable Cross Section with Loose Banks and Constant Slope

Two assumptions are made herein. The first is that the friction factor varies as –0.28 power of the discharge (p = -0.28) and the second is that b = 2f. The assumption that b = 2f is supported by stable river and canal data (Neill, 1973), where b is about 0.55 and f is about 0.26 which validates the assumption. The ratio b/f = 2 is suggested as a limiting or maximum value for channel adjustments of straight reaches. Adopting the two assumptions and using Equation 23, it follows that f = 0.25, b = 0.5 and m = 0.25. The data of 16 stations (Williams, 1978) show field values of f = 0.26, b = 0.54 and m = 0.21 which supports the hypothesis of extremal energy slope. It can be shown that (Hafez 2000) Williams, using the minimum variance theory, obtained values of f = 0.3, b = 0.48 and m = 0.22.

It should be noted that for channel cross sections with constant slope, identical values of the hydraulic exponents are obtained by the extremal energy slope and the Darcy-Weisbach formula for the friction factor when using the same assumption about the variation of the friction factor with the discharge. It follows then that channel cross sections with constant slope are at a state of minimum energy slope under long term equilibrium conditions.

SUMMARY OF RESULTS

In the following is a summary of the applications of the empirical function and extremal energy slope and stream power along with field data and the minimum variance theory values of the hydraulic exponents for comparison purposes.

Table 1. Values of at-a-Station Hydraulic Exponent for Depth, f

Case	Field Data	The Minimum Variance	The Empirical Function	Extremal Energy Slope (Hafez)
Stable Cross Section with Constant Width and Slope (b < 0.03)	0.46-0.52	0.5	0.5	0.5
Stable Cross Section with Constant Slope and Constrained Width (b=0.55f)	0.4	0.42	0.41	0.42
Stable Cross Section with Constant Slope and Free Width (b=f)	0.41	-	0.36	0.36
Stable Cross Section with Constant Slope and Firm Banks (b=0.08)	0.5	0.54	0.47	0.52
Stable Cross Section with Constant Slope and Firm Banks (b=0.19f)	0.5	0.53	0.47	0.51
Stable Cross Section with Constant Slope and Firm Banks (b=0.12-0.06f)	0.5	0.52	0.47	0.51
Stable Cross Section with Constant Slope and Loose Banks (b=2f)	0.26	0.3	0.29	0.25

Table 2. Values of at-a-Station Hydraulic Exponent for Velocity, *m*

Case	Field Data	The Minimum Variance	The Empirical Function	Extremal Energy Slope (Hafez)
Stable Cross Section with Constant Width and Slope (b<0.03)	0.47-0.5	0.5	0.5	0.5
Stable Cross Section with Constant Slope and Constrained Width (b=0.55f)	0.34	0.35	0.36	0.35
Stable Cross Section with Constant Slope and Free Width (b=f)	0.18	-	0.28	0.28
Stable Cross Section with Constant Slope and Firm Banks (b=0.08)	0.42	0.38	0.45	0.4
Stable Cross Section with Constant Slope and Firm Banks (b=0.19f)	0.42	0.36	0.44	0.39
Stable Cross Section with Constant Slope and Firm Banks (b=0.12-0.06f)	0.42	0.39	0.44	0.4
Stable Cross Section with Constant Slope and Loose Banks (b=2f)	0.21	0.22	0.13	0.25

Table 3. Values of at-a Station Hydraulic Exponent for Width, *b*

Case	Field Data	The Minimum Variance	The Empirical Function	Extremal Energy Slope (Hafez)
Stable Cross Section with Constant Width and Slope (b < 0.03)	0.01-0.05	0.0	0.0	0.0
Stable Cross Section with Constant Slope and Constrained Width (b=0.55f)	0.26	0.23	0.23	0.23
Stable Cross Section with Constant Slope and Free Width (b=f)	0.41	-	0.36	0.36
Stable Cross Section with Constant Slope and Firm Banks (b=0.08)	0.08	0.08	0.08	0.08
Stable Cross Section with Constant Slope and Firm Banks (b=0.19f)	0.08	0.11	0.09	0.10
Stable Cross Section with Constant Slope and Firm Banks (b=0.12-0.06f)	0.08	0.09	0.09	0.09
Stable Cross Section with Constant Slope and Loose Banks (b=2f)	0.54	0.48	0.58	0.50

Table 4. Values of Hydraulic Exponents for Downstream Geometry

	b	f	m	z
Field Data	0.5	0.4	0.1	-0.75
Minimum Variance Theory	0.53	0.37	0.1	-0.73
The Empirical Function	0.47	0.42	0.11	-0.77
Extremal Stream Power (Hafez)	0.49	0.42	0.09	-0.76

SUMMARY AND CONCLUSIONS

The following observations can be stated:

1. Successful predictions of the hydraulic exponents (b, f, m and z) in the hydraulic geometry regime type equations is possible using an empirical function and extremal methods of energy slope and stream power.

2. For channel cross sections with firm and stable banks and constant slope, one degree of freedom exists by changing the roughness. Equable change between the velocity and depth exponents occurs ($m = f = 1/2$) and the friction factor varies as the –0.5 power of discharge according to flume and field data and the minimum variance theory. The empirical function gives $m = f = 1/2$ and the extremal energy slope similarly gives m = f = 1/2. This case is suitable for stable river sections with firm banks with stiff clay or protected by riprap.

3. Channel cross sections with constant slope and banks that are less firm than in the previous case might be constrained by the relation $b = 0.55f$. Field data of Leopold and Maddock (1953), using 20 cross sections, gives $b = 0.26$, $f = 0.4$ and $m = 0.34$ and the minimum variance theory gives $b = 0.23$, $f = 0.42$ and $m = 0.35$. The empirical function gives $b = 0.23$, $f = 0.41$ and $m = 0.36$. Extremal energy slope gives $b = 0.23$, $f = 0.42$ and $m = 0.35$. The friction factor is supposed to vary as the –0.28 power of discharge

4. For river reaches where the width is completely free, it is assumed that $b = f$. Yang et al. (1981) used this assumption and obtained b = f = 0.41 and proved that these are the limiting values of field data by Barr et al. (1980). Slope varied as –2/11 power of discharge. The empirical function gives $b = f = 0.36$ but under constant slope assumption. Assuming constancy of the friction factor with discharge yields $b = f = 0.4$ and $m = 0.2$ according to the extremal energy slope which agrees very well with Yang's predictions. This example demonstrates the equivalency between the minimum energy slope and the maximum friction factor concepts. This case is suitable for reaches where the banks are loose and bank erosion is at maximum.

5. For downstream increase in discharge, the channel adjusts its slope (longitudinal profile), depth and width. Field data of Leopold and Maddock (1953), gives $b = 0.5$, $f = 0.4$ and $m = 0.1$. The minimum variance theory gives $b = 0.53$, $f = 0.37$, $m = 0.1$ and slope (profile) to vary as –0.73 (i.e. z = -0.73) power of discharge. The empirical function gives $b = 0.47$, $f = 0.42$, $m = 0.12$ and $z = -0.77$. Extremal energy slope gives $b = 0.49$, $f = 0.42$, $m = 0.09$ and $z = -0.76$. This case is suitable for reaches where flow increases in the downstream direction due to incoming tributary flows.

6. For channel sections with constant slope, values of $b = 0.08$, $f = 0.5$ and $m = 0.42$ were reported by Williams (1978) using 74 stations. Based on field data three relations of width were found possible, namely $b = 0.19f$, $b = 0.12\text{-}0.06f$ and $b = 0.08$. Extremal energy slope gives $b = 0.1$, $f = 0.51$ and $m = 0.39$ based on $b = 0.19 f$. For $b = 0.12\text{-}0.06 f$, it gives $b = 0.09$, $f = 0.51$ and $m = 0.4$. For $b = 0.08$, it gives $f = 0.52$ and $m = 0.4$.

7. For channel sections with constant slope and free width to change, it is assumed that $b = 2 f$. Values of $b = 0.54$, $f = 0.26$ and $m = 0.21$ were reported by Williams (1978) using 16 stations. Extremal energy slope gives $b = 0.5$, $f = 0.25$ and $m = 0.25$.

8. Channel cross sections under long term equilibrium are at the state of extremal (minimum) energy slope. This is supported by field observations in which these channels have constant slope. In addition, river reaches under long term equilibrium are at the state of extremal (minimum) stream power. This is equivalent to the constancy of the stream power along the river channel reach.

RECOMMENDATIONS

Further application of the empirical function, extremal energy slope and extremal stream power is recommended in addition to other extremal methods as discussed by Hafez (2000). Field data for different reaches of the river are needed with accurate evaluation of bank conditions. The importance of bank strength on the adjustments in the depth, width, velocity and slope is shown.

The hydraulic exponents are useful indicators of the response of the river to altered conditions of discharge and sediment loads and to existing and future structures. In addition they are useful for predicting the average water depth and velocity, especially for high flow releases downstream. The analysis presented in this paper could be applied as well to the investigation of maintenance and flushing flows in canals and open drains.

ACKNOWLEDGMENTS

The writer would like to thank greatly Prof. Dr. Mohamed El-Korany, The Nile Research Institute, Egypt, for his excellent review of this paper.

REFERENCES

Barr, D. I.H., M.K. Alan and A. Nishat; "A Contribution to Regime Theory Relating Principally to Channel Geometry", Proc. Inst. Civ. Eng., 69, 651-670, 1980.

Blench, T.; "Regime Theory Design of Canals with Sand Beds", Journal Irrig. Drainage Div., ASCE, 96 (IR2), pp. 205-213, June 1970.

Blench, T.; "Regime Theory for Self-formed Sediment-Bearing Channels", Trans. ASCE, 117, pp. 383-408, 1952

Chang, H.H.; "Fluvial Processes in River Engineering", John Wiley & Sons, Inc., 1988.

Davies, T.R.H. and A.J. Sutherland; "Extremal Hypotheses for River Behavior", Water Resources Research, Vol. 19, No.1, pp.141-148, Feb. 1983.

Hafez, Y.I.; "Response Theory for Alluvial River Adjustments to Environmental and Man-Made Changes", Journal of Environmental Hydrology, paper 14, Volume 8, October 2000.

Lacey, G.; "Stable Channel in Alluvium", Proc. Inst. Civ. Eng., London, 229, 1930.

Lacey, G.; "Uniform Flow in Alluvial Rivers and Canals", Proc. Inst. Civ. Eng., London, 9; discussion,11, 1958.

Langbein, W.B., "Geometry of River Channel", Journal Hydraul. Div. ASCE, 90 (HY2), pp.301-312, March 1964.

Leopold, L.B., and T. Maddock Jr.; "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications", USGS Professional paper 252, 1953.

Mackin, J.H.; "Concept of the Graded River", Geol. Soc. Am. Bull., 59, pp. 463-512, May 1948.

Neill, C.R.; "Stable Sand bed Rivers", NRC Proc. Hydrology Symposium, Edmonton, Canada 1973.

Nile Research Institute; "River Regime of the Nile in Egypt, 1992.

Nizery, A., and G. Braudeau; "Discussion of "Design of Stable Channel" by E.W. Lane, Transactions, ASCE, Vol. 120, pp. 1266-1269, 1955.

Robbins, C.H., and A. Simon; "Man Induced Channel Adjustment in Tennessee Streams", US Geological Survey Water-Resources Investigations report 82-4098, Nashville, TN 1983.

Simons, D.B., and M.L. Albertson; "Uniform Water Conveyance Channels in Alluvial Material", Journal Hydraul. Div. ASCE, 86 (HY5), pp. 33-71, May 1960.

Simons, D.B., E.V. Richardson, and M.L. Albertson; "Flume Studies using Medium Sand", Water Supply Paper 1498 A, Geol. Survey, U.S. Dept. of the Interior, Washington, D.C., 1961.

Williams, G.P.; "Hydraulic Geometry of River Cross Sections", Geol. Survey professional paper; 1029, 1978.

Yang, C.T.; "Energy Dissipation Rate Approach in River Mechanics", Sediment Transport in Gravel Bed Rivers, Edited by C.R. Thorne, J.C. Bathurst and R.D. Hey, John Wiley & Sons Ltd., 1987.

Yang, C.T.; "Minimum Unit Stream Power and Fluvial Hydraulics", Journal of the Hydraul. Div., ASCE, Vol. 102, No. HY7, July 1976.

Yang, C.T.; "Dynamic Adjustment of Rivers", Third International Symposium on River Sedimentation, The University of Mississippi, March 31-April 4, 1986.

Yang, C.T., and C.S. Song; "Theory of Minimum Rate of Energy Dissipation", Journal of the Hydraul. Div., ASCE, Vol. 105, No. HY7, July 1979.

Yang, C.T., C.S. Song, and M.J. Woldenberg; "Hydraulic Geometry and Minimum Rate of Energy Dissipation", Water Resources Research, Vol. 17, No. 4, pages 1014-1018, Aug. 1981.

ADDRESS FOR CORRESPONDENCE
Youssef I. Hafez
The Nile Research Institute, National Water Research Center
El_Kanater, Delta Barrage,
Cairo 13261
Egypt

E-Mail youssef_hafez@usa.net.