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ANALYTICAL SOLUTIONS FOR WELL DRAWDOWN WITH WELL LOSSES
2. REAL WELL NEAR BOUNDARY - SOLUTION BY IMAGE WELL

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The solutions in the preceding Paper 1 were based on the assumption of an infinite aquifer. This assumption, however, is not valid in the case where a well is located near an infinite boundary, which may be impermeable or have different permeability, and where the boundary is within the drawdown cone of the punping well. A solution is presented for drawdown in a pumping well that includes well skin losses and an infinite boundary. The solution is based on the theory of mirror reflection of image wells.

INTRODUCTION

The influence of an infinite impermeable boundary is solved by the application of the mirror reflection theory. We assume that the well drawdown during the discharge from the well is identical with the influence resulting from the superposition of the effect of discharge, Q, from the given real well and the effect of simultaneous discharge of the same Q from an imaginary well. This image well is a mirror image of the real well beyond the impermeable boundary at the same distance from the impermeable boundary as the real well.

THEORY

Real Well in the Proximity of Lateral Impermeable Boundary

With the discharge Q of a well in the proximity of a lateral impermeable boundary, which is "within reach" of the well, the resulting ground water drawdown, s, is determined by the sum of the theoretical drawdown, s_{re}, due to the discharge from the real well and the drawdown, s_f, due to the discharge from the image well placed in a mirror-reflected position.

$$s = s_{re} + s_f \tag{1}$$

for the real well

$$s_{re} = s_{te} + s_{SKIN} \tag{2}$$

where

s_{te} = drawdown at an "ideal" well (W = 0)

s_{SKIN} = additional drawdown at a well caused by additional resistance or well loss.

If we consider the non-stationary flow regime, the water level drawdown in the real well is given by solving the equation in dimensionless form (Agarwal et al., 1970):

$$\frac{\partial^2 s_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial s_D}{\partial r_D} = \frac{\partial s_D}{\partial t_D} \tag{3}$$

The complete solution of Equation (3) may be obtained by the application of the Laplace transform. The Laplace transform is inverted numerically using the Stehfest algorithm 368 (Wei Chun Chu, 1980; Pech, 1986):

$$s_V = \frac{Q}{2\pi T} \sum_{i=1}^{n} con(j,k) \sum_{i=0}^{n} \binom{m}{i} (-1)^i \cdot$$

$$\left[\frac{K_0\left(c^{1/2}\right) - W c^{1/2} K_1\left(c^{1/2}\right)}{c^{3/2}\left[c^{1/2} K_1\left(c^{1/2}\right) + C_D c^{1/2}\left(K_0\left(c^{1/2}\right) + W c^{1/2} K_1\left(c^{1/2}\right)\right)\right]} + \frac{K_0\left(r_D c^{1/2}\right)}{c^2 K_1\left(c^{1/2}\right)} \right] \tag{4}$$

where

T = transmissivity; S = storativity; Q = well discharge; C_D = a dimensionless well bore storage constant (Ramey, 1970); c = the variable in Laplace space ; $r_D = 2x_0/r_W$; x_0 (see Figure 1),

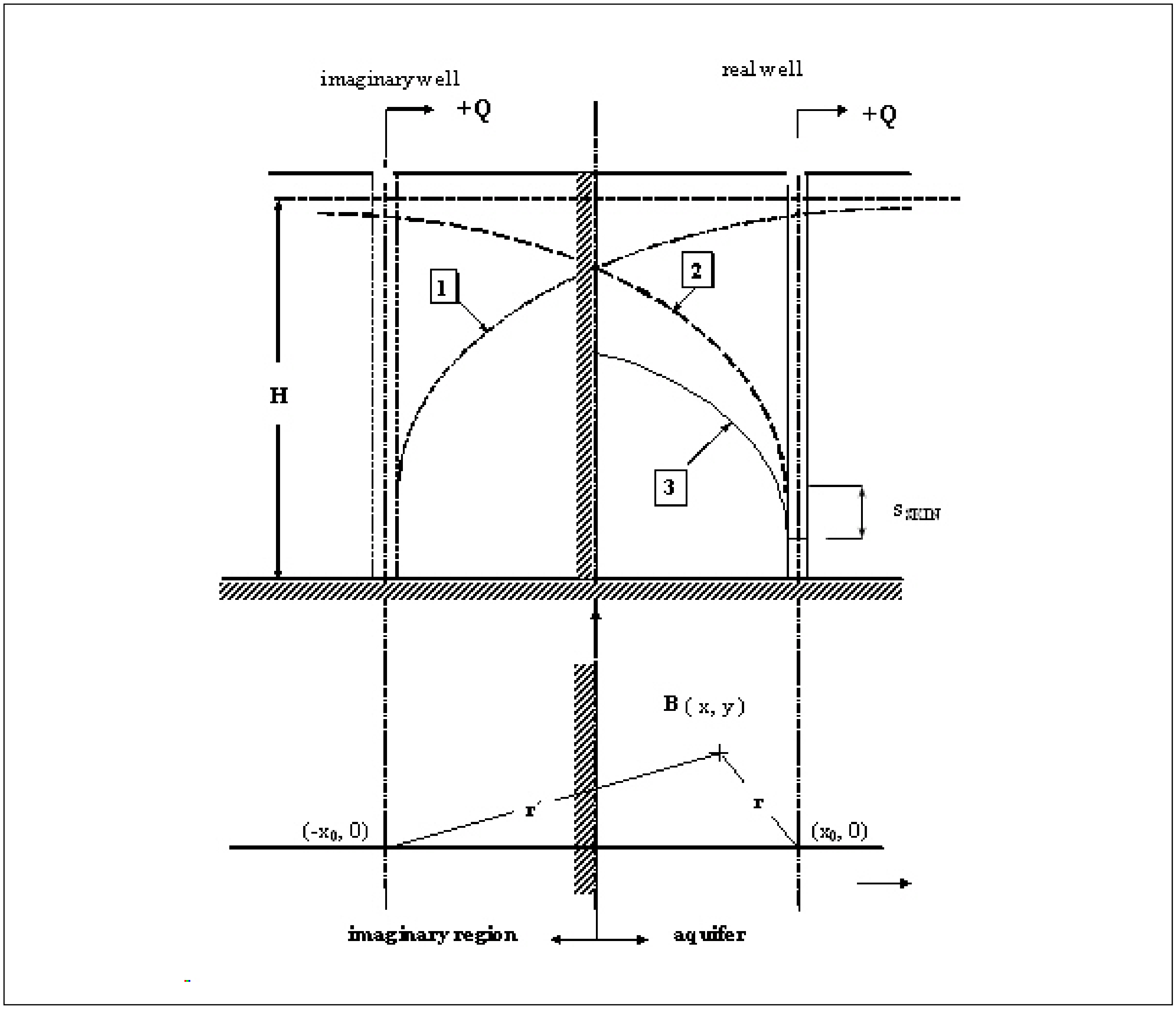

Figure 1. Schematic drawing of the well in the proximity of impermeable lateral boundary (1 - drawdown due to imaginary well, 2 - drawdown due to the real well, 3 - overall drawdown.

r_w = well radius; W = skin factor; $K_0(x)$ is a Bessel function of the second kind of order zero; $K_1(x)$ is a Bessel function of the second kind of order one; n= 10; and

$$con(j,k) = \frac{(-1)^{j-1}}{k}\binom{k}{j} j\, m^{k-1} \frac{\ln 2}{t_D\, r_D} \frac{(2m)!}{m!\,(m-1)!} \tag{5}$$

where

t_D = dimensionless time ($Tt/r_w^2 s$)

or, for $t_D < 25$, i.e. in the condition when the Jacob semilogarithmic approximation of the Theis well function is applicable, it is

$$s_V = \frac{Q}{2\pi T}\left[\ln\frac{2{,}246Tt}{r_W{}^2 S} + W + \ln\frac{2{,}246Tt}{r'^2 S}\right] \tag{6}$$

If $r = 0$, then $r' = 2x_0$. (see Figure 1).

After modification, the drawdown in the well is expressed by the relation

$$s_V = \frac{Q}{\pi T}\left[\ln \frac{2{,}246 T t}{r_W (2x_0) S} + \frac{1}{2} W\right] \tag{7}$$

The drawdown in any point B determined by the coordinates x, y (Figure 1) for a non-stationary flow regime, is

$$s_B = \frac{Q}{2\pi\ T} \sum_{i=1}^{n} con(j, k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \left\{ \frac{K_0\left(r_D c^{1/2}\right) + K_0\left(r_D{}' c^{1/2}\right)}{c^2\ K_1\left(c^{1/2}\right)} \right\} \tag{8}$$

where

$r_D = r/r_W$ and $r_D{}^t = r_t / r_W$

The distances r and r′ are determined from the relations

$$r' = \sqrt{(x + x_0)^2 + y^2} \tag{9}$$

$$r = \sqrt{(x - x_0)^2 + y^2} \tag{10}$$

If we consider the application of the Jacob semilogarithmic approximation, the water level drawdown in point B is

$$s_B = \frac{Q}{2\pi\ T}\left[ln\ \frac{2{,}246\ T\ t}{r^2\ S} + ln\ \frac{2{,}246\ T\ t}{r'^2\ S}\right] \tag{11}$$

or, for the sake of simplification,

$$s_B = \frac{Q}{\pi\ T}\left[ln\ \frac{2{,}246\ T\ t}{r r' S}\right] \tag{12}$$

Well in the Proximity of Lateral Constant Head Recharge Boundary

By way of example let us use a well in the proximity of a constant head recharge boundary (surface flow) fully penetrating the aquifer. The solution is analogous with that applied to the impermeable boundary. The difference is that in the case of an imaginary well we consider constant injection Q (it is an injection well). Overall drawdown produced by the discharge Q from the real well is determined as the sum of the drawdown in the real well and the negative drawdown due to injection Q into the imaginary well.

Analogous to the impermeable lateral boundary (Equation 1) the overall drawdown is

$$s = s_{re} - s_f \tag{13}$$

The drawdown in the discharge well is given by Equation 3.

In case of application of the Jacob semilogarithmic approximation, the drawdown in the discharge

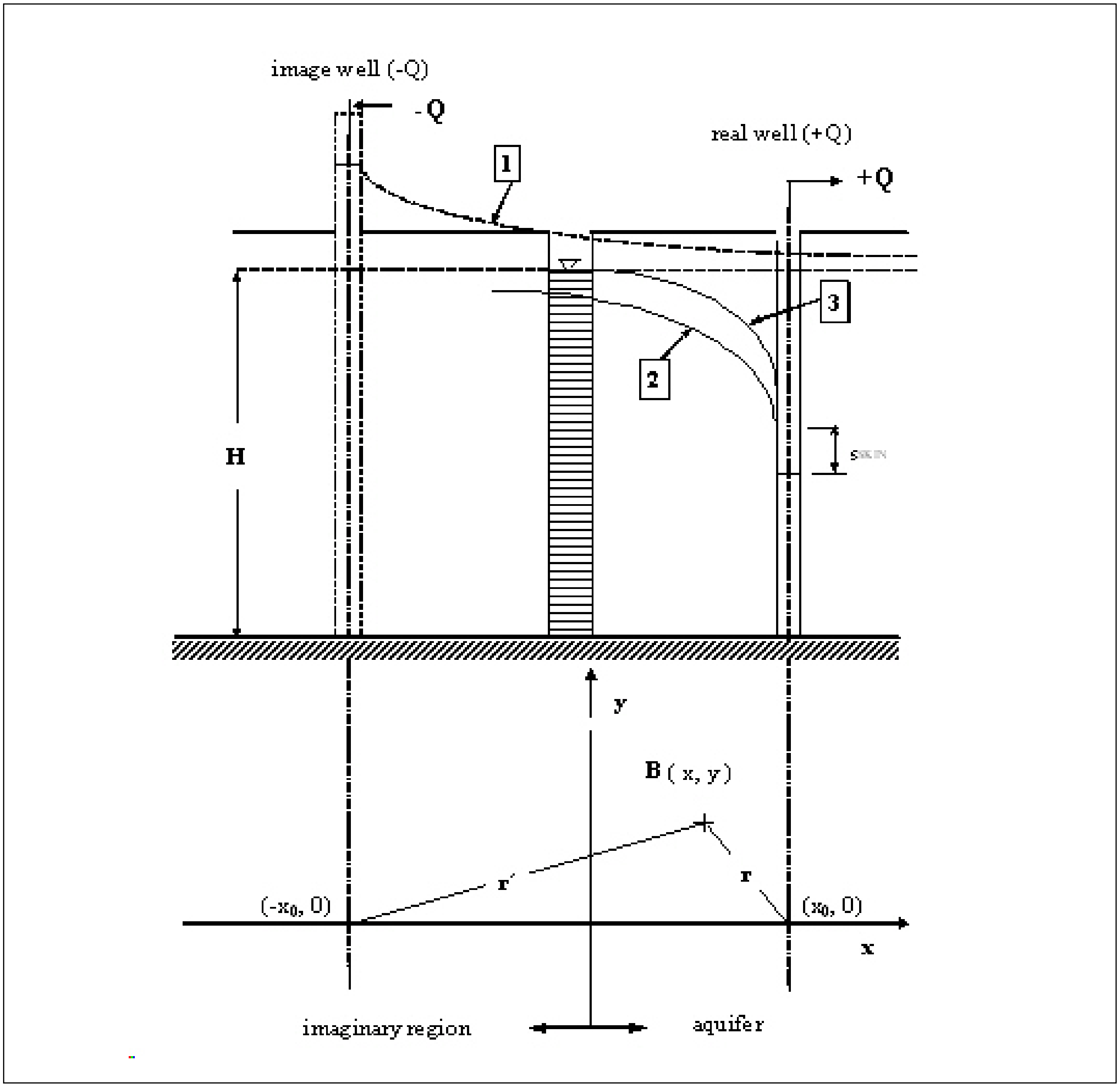

Figure 2. Schematic drawing of the well in the proximity of lateral feeding boundary (1 – drawdown due to imaginary well, 2 – drawdown due to real well, 3 – overall drawdown).

well is

$$s_V = \frac{Q}{2\pi T}\left[\ln\frac{2{,}246Tt}{{r_W}^2 S} + W - \ln\frac{2{,}246Tt}{r'^2 S}\right] \tag{14}$$

or, after modification,

$$s_V = \frac{Q}{2\pi T}\left[\ln\frac{r'}{r_W} + W\right] \tag{15}$$

The drawdown, s_V, does not depend on time and the curve, $s_V = \mathrm{f}(\log t)$, proceeds horizontally, which means that the influence of the lateral boundary will manifest itself after certain time by the stabilization of the drawdown. The transmission capacity and storage capacity computations can be based only on the initial part of the inflow test before the part which can be evaluated by the Jacob semilogarithmic approximation is reached.

The drawdown in any point B (analogous to the case of impermeable boundary) is

$$s_B = \frac{Q}{2\pi\ T} \sum_{i=1}^{n} con\left(j,k\right) \sum_{i=0}^{m} \binom{m}{i} \left(-1\right)^i \left\{ \frac{K_0\left(r_D\, c^{1/2}\right) - K_0\left(r_D{}'\, c^{1/2}\right)}{c^2\ K_1\left(c^{1/2}\right)} \right\} \tag{16}$$

and, when considering the applicability of the Jacob semilogarithmic approximation,

$$s_B = \frac{Q}{2\pi\ T}\left[ln\ \frac{2{,}246\ T\ t}{r^2\ S} - ln\ \frac{2{,}246\ T\ t}{r'^2\ S} \right] \tag{17}$$

or, after modification,

$$s_B = \frac{Q}{2\pi\ T}\left[ln\ \frac{r'}{r} \right] \tag{18}$$

CONCLUSION

In this contribution the relations are derived for the determination of the drawdown in a well with well losses situated in the proximity of the permeable or impermeable boundary. By means of the derived relationships we can determine drawdown at an arbitrary place when the real well, with additional resistance and wellbore storage taken into account, is situated near boundary.

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