The term hydraulic geometry connotes the relationships between the mean cross-section geometry (width, depth, cross-section, meander length), and the hydraulic variables which include the mean slope, mean friction, and mean velocity for a given influx of water and sediment to the channel and the specified channel boundary conditions at any cross-section and downstream. The hydraulic geometry relations are of great practical value in water river temperature modeling. Leopold expressed the hydraulic geometry relationships for a channel in the form of power functions of discharge. The Tigris River, which has a length of 1800 km is one of the two main rivers in Iraq. To describe its hydraulic geometry, a river reach of 202.5 km from Mosul to Bejee was surveyed using Leopold’s method at maximum, minimum and average discharges. Manning’s n was also calculated and the effect of these discharges on width to depth ratio was shown at all these discharges. The results show that there are different hydraulic geometry characteristics along the river reach with high width to depth ratio, especially at minimum discharges, and this has affected river water temperature.
INTRODUCTION

The term hydraulic geometry connotes the relationships between the mean stream channel form and discharge both at-a-station and downstream along a stream network in a hydrologically homogeneous basin. The channel form includes the mean cross-section geometry (width, depth, cross-section, meander length), and the hydraulic variables which include the mean slope, mean friction, and mean velocity for a given influx of water and sediment to the channel and the specified channel boundary conditions (Singh, 2003).

The hydraulic geometry relations are of great practical value in prediction of channel deformation, layout of river training works, design of stable canals and intakes, river flow control works, irrigation schemes, river improvement works, and so on. Richards (1976) has reasoned that hydraulic geometry relations through their exponents can be employed to discriminate between different types of river sections. It is also important in planning for resource, impact assessment and river temperature modeling. (Allen et al. 1994; Singh, 2003).

Importance of hydraulic geometry in river water temperature modeling

Analysis of water temperature regimes in rivers has lately taken on added importance, primarily for economic reasons. Bartholow (1989) gives the essential components of hydraulic geometry in river water temperature modeling:

Stream Width: Stream width can be a very sensitive parameter in modeling water temperatures. All of the heat flux takes place at either the air-water interface or the water-ground interface, both of which are as wide as the wetted stream width. Also, the ratio of width/depth has an important influence on the sensitivity of water temperature to the flux of heat.

Manning’s n: This is a measure of the roughness of the streambed and channel, which causes flowing water to backup due to friction, and is a necessary component in predicting daily maximum water temperatures. At lower flows, the roughness tends to be due primarily to the stream bottom characteristics; as the flow increases, the whole channel shape, including river bends and constrictions, become dominant. Therefore, Manning’s n is not constant with changing flow.

Travel Time: Travel time is an alternative to Manning’s n. Travel time is the inverse of velocity. If velocity is measured in units of length per time, then travel time is measured in units of time per length, such as seconds per kilometer. Stream velocity, and therefore time of travel, vary with discharge.

Mathematical relationships

Although some concepts of hydraulic geometry were proposed toward the end of the nineteenth century, the real impetus toward formulating a theory of hydraulic geometry was provided by the work of Leopold and Maddock (1953). This work has been applied on many rivers for water temperature modeling around the world by Singh (2003), Howard and Pelletier (2002), Bartholow (2002), Payne (2002), Bashar and Gulliver (2001) and others.

Leopold and Maddock expressed the hydraulic geometry relationships for a channel in the form of power functions of discharge as:

\[ B = aQ^b, \quad d = cQ^f, \quad V = kQ^m \]  

(1)
where $B$ is the channel width; $d$ is the flow depth; $V$ is the flow velocity; $Q$ is the flow discharge; and $a$, $b$, $c$, $f$, $k$, and $m$ are parameters. To Equation 1, also added:

$$n = NQ^p, S = sQ^y$$  \hspace{1cm} (2)

where $n$ is Manning’s roughness factor; $S$ is slope; and $N$, $p$, $s$, and $y$ are parameters. Exponents $b$, $f$, $m$, $p$ and $y$ represent, respectively, the rate of change of the hydraulic variables $B$, $d$, $V$, $n$ and $S$ as $Q$ changes and coefficients $a$, $c$, $k$, $N$ and $s$ are scale factors that define the values of $B$, $d$, $V$, $n$ and $S$ when $Q = 1$.

The hydraulic variables, width, depth and velocity, satisfy for rectangular channels the continuity equation:

$$Q = BdV$$  \hspace{1cm} (3)

the coefficient and exponents in Equation 1 satisfy

$$ack = 1, b + m = 1$$  \hspace{1cm} (4)

**Determination of at-a-station hydraulic geometry:**

The hydraulic geometry at a given cross-section, which is called the at-a-station hydraulic geometry, can be determined by these steps:

1. Get several sets (three or more measurements are much better than two) of discharge measurements together with the water width, mean depth, and mean velocity at the time of the measurement.

2. Take the natural log of both width (or mean depth, mean velocity) and discharge and perform a standard linear regression with discharge being the independent variable. Be sure to use consistent units. The antilog of the intercept should be computed, not forced to zero, because it will be equal to the “constant” term in the relationship. The “exponential” term will be the coefficient (slope) of the regression; the antilog of “$b$” should not be taken because it is a unitless term.

3. Check to see if the values for $b$, $f$, and $m$ add up to 1 and that the product of values of $a$, $c$, and $k$ equals 1.

4. Manning’s equation is commonly used to estimate depth ($d$) from flow ($Q$), as in Equation 5 with Manning’s roughness coefficient ($n$), width ($w$), and slope ($S$), assuming the hydraulic radius equals the depth and the width is large compared to the depth (Lindeburg, 1987; Howard and Pelletier, 2002; metric units):

$$d = [(n*Q) / (S^{0.5}*w)]^{0.6}$$  \hspace{1cm} (5)

If the flow ($Q$), width ($w$), and the depth ($d$) are known, then the continuity equation can be used to estimate velocity ($v$):

$$v = Q / (w*d)$$  \hspace{1cm} (6)

**Study area**

The Tigris River has a length of 1800 km. It is one of the two main rivers in Iraq. It has five tributaries inside Iraq, they are from north to south, Al-Khaboor river, Big Zaab (upper), Small Zaab
(lower), Al-Authem, and Dyalla river. The Tigris River supplies agricultural and industrial production, domestic usage, power generation and recreational activities. The Tigris, which is one of the longest rivers in the world, has its basin in four countries (Turkey, Syria, Iran and Iraq). It suffers the adverse effects of the control of flow, especially discharge which is changing continuously. The elevation of water in the river also changes and, as a result, intakes of water treatment plants and power generation plants are affected (Al-Obaidy, 1996; Al-Jubori, 1998; Al-Naish, 1999).

The river reach of interest has a length of 202.5 km, and extends from Mosul at km 177.5 in the north, downstream to km 380 near Bejee at the Al-Fat-haa bridge, these distances are measured from km zero at Feshkhabor where the river first enters Iraq. There are three main gauging stations along this distance, one at Mosul (for measuring the discharge from Mosul dam plus the discharge from the catchment area between the dam to the city), another at Al–Gayaraa which gives the sum of discharges from the Tigris River and the upper Zab River, and a third at Al-Fat-haa where the total discharge of lower Zab and Tigris River discharge upstream is measured. This study also uses data measured at some cross-sections along the river in Mosul. Figure (1) shows segmentation of the Tigris River with the locations of gauging stations.

Purpose

With the present shortage of field data, this study will be an approximation to identify the hydraulic geometry of the 202.5 km length of Tigris river reach from Mosul in the north of Iraq.

Figure 1. Schematic of Tigris River segmentation with gauging stations from Mosul to Beje.
to Bejee downstream, using available data. This information can be used to address any river engineering and environmental problems with an emphasis on the modeling of river water temperature.

**PROCEDURE**

**River flow data**

Among the three gauging stations there are only sufficient data for the station at Mosul. These data were taken as a start point to get the other flow data required for the other two stations using these steps:

1. All the daily flow data available for three years (2000, 2001, 2002) was collected from the water resources office in Mosul.

2. The maximum, minimum and average flow for each month were calculated using these data.

Table 1. Percentage of seasonal feeding of Tigris River tributaries in Tigris River flow from (1961-1996).

<table>
<thead>
<tr>
<th>Section No.</th>
<th>slope</th>
<th>Remarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000404</td>
<td></td>
<td>Al-Naish, 1999</td>
</tr>
<tr>
<td>2</td>
<td>0.000404</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.000404</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.000676</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.000544</td>
<td>Using data in Table 5-1, the slope is the difference between elevations of Tigris river for average flow at each station divided by the actual distance between them</td>
<td>Al-Obaidy, 1996</td>
</tr>
<tr>
<td>6</td>
<td>0.00058</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adopted from (Al-Jubori, 1998).
March is the first month of spring.

3. As there are not sufficient data for Al-Gayaraa and Al-Fat-haa stations, the percentage of the total discharge at Al-Fat-haa, for the Tigris River in Mosul, Upper Zab and Lower Zab are shown in Table 1 (Al-Jubori, 1998). This information was taken to calculate the maximum, minimum and average for each month at Al-Gayaraa and Al-Fat-haa stations depending on that at Mosul station.

The reach of interest of the Tigris River needs to include five segments according to the available data at each location as shown in Figure 1 and Table 2.

**Calculation of Leopold coefficients**

1. For each cross-section, wetted width, depth, and velocity as a function of flow, was estimated from recorded data available at water resources offices, and three readings at each section are used.

2. For sections which are not gauging stations, the relation between flow and water surface elevation for certain cross-section drawn at a certain scale (provided also by the water resources office), has been used to calculate the area of flow and wetted width. Average depth is the result...
of dividing area by width and velocity is the result of dividing flow by area and three discharges were used at each section.

3. Using the MATLAB program for curve fitting gives the best values of Leopold coefficients for each condition.

**Calculation of hydraulic geometry**

1. The hydraulic geometry at maximum, minimum and average flows was estimated from available data for each section applying Leopold relationships obtained in the previous step. At sections 1, 2, 3, and 4, flow data at Mosul was taken, for section 5 flow data at Al-Gayaraa was used and for section 6 flow data at Al-Fat-haa was employed.

2. Manning’s coefficient for each section at minimum, maximum and average discharge has been calculated by Equation 7.

\[
v = \frac{1}{n} \frac{2}{R^\frac{1}{3}} S^\frac{1}{2}
\]  

(7)

Table 2. Tigris River segmentation.

<table>
<thead>
<tr>
<th>River segment No.</th>
<th>Distance (Km from Feshkhaboor)</th>
<th>Upstream, downstream Section No.</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>177.5-178.5</td>
<td>1,2</td>
<td>□ Section 3 is at Mosul station</td>
</tr>
<tr>
<td>2</td>
<td>178.5-188</td>
<td>2,3</td>
<td>□ Section 5 is at Al-Gayaraa station</td>
</tr>
<tr>
<td>3</td>
<td>188-195.5</td>
<td>3,4</td>
<td>□ Section 6 is at Al-Fat-haa station</td>
</tr>
<tr>
<td>4</td>
<td>195.5-238</td>
<td>4,5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>238-380</td>
<td>5,6</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Slope of Tigris River at each section.

<table>
<thead>
<tr>
<th>Section No.</th>
<th>slope</th>
<th>Remarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000404</td>
<td></td>
<td>Table 1-5</td>
</tr>
<tr>
<td>2</td>
<td>0.000404</td>
<td></td>
<td>Al-Naish, 1999</td>
</tr>
<tr>
<td>3</td>
<td>0.000404</td>
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<td>4</td>
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</tr>
<tr>
<td>6</td>
<td>0.00058</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
where \( v \) is the velocity (m/sec), \( R \) is the hydraulic radius (m), which equals the cross-section area divided by the wetted perimeter or the width if the width is large compared to depth, and \( S \) is the slope. The slope of the Tigris River at each section is given in Table 3.

3. Width to depth ratio for each section at maximum, minimum and average monthly flow was calculated.

RESULTS

1. Figure 2 shows the minimum, maximum and average discharges along the reach of interest at each month of the year

2. Leopold coefficients for each section are shown in Table 4.

3. Figures 3, 4, and 5 show the relationship between Manning’s \( n \) and discharge at each section for maximum, minimum and average flow.

4. Width to depth ratio is shown in Figures 6, 7 and 8.
5. Using Equations 5 and 6 for any width and discharge of the river at any section, the depth and the velocity can be estimated.

6. For a certain discharge the average area of upstream and downstream multiplied by the distance gives the volume of water in the river which is very important in water temperature modeling.

DISCUSSION AND CONCLUSIONS
It can be concluded that:

1. Leopold coefficients describe the characteristic of hydraulic geometry of the Tigris River.
2. Width, depth and velocity at any discharge, Figure 2, can be computed at each section.
3. Figures 3, 4, and 5 show that each section of the river has its special friction characteristics and Manning’s n related to certain discharge.
4. Sections 1, 2, and 6 show a small effect of increasing discharge at maximum, minimum and
average discharges on Manning’s n.

5. Sections 3, 4, and 5 show reasonable effect of increasing discharge at maximum, minimum and average discharges on Manning’s n.

6. It is evidence from Figures 6, 7, and 8 that the Tigris River has a high width to depth ratio over all the year (25-591) at maximum flows, (28-1647) at minimum flows and (28-883) at average flow. This has a great influence on the energy budget of the river, especially at minimum discharges.

7. For section 1 the minimum width to depth ratio was 293 in May during maximum discharges while the maximum value is 1505 in March during minimum discharges. This is because it is a wide shallow section.

8. Section 2, which has a more regular shape, has minimum width to depth ratio 31 in March during minimum discharges, with a maximum value 201 in April during maximum discharges.

9. Section 3 has minimum value of width to depth ratio 53 in April during maximum discharges and maximum value of 129 in March during minimum flows. This section is at the gauging station at Mosul with constant width.

10. Section 4, which is a reasonably deep section with almost constant width, has minimum width to depth ratio 40 in April during maximum flows and maximum value of 67 in March during minimum discharges.

11. Section 5 has minimum width to depth ratio 26 in April during maximum discharges and maximum value 35 in March during minimum discharges. This is a deep section.

12. Section 6 has 25 minimum width to depth ratio at maximum discharge in April and maximum value of 103 in March during minimum discharges.

13. For section 2, only the values of width to depth ratio at average flows is greater than that at minimum flows and less than that at maximum flows, while this relation is the reverse in the other sections.

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