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FITTING PROBABILITY DISTRIBUTION FUNCTIONS TO RESERVOIR INFLOW AT HYDROPOWER DAMS IN NIGERIA

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Some methods of probability distribution analysis were evaluated for the prediction of reservoir inflow at hydropower dams in Nigeria: The hydropower dams include Kainji, Shiroro and Jebba. The reservoir inflow data were subjected to probability distribution analysis including Gumbel, normal, log-Pearson type III, and log-normal. The selection of the appropriate probability distribution model for each hydropower dam was based on goodness of fit tests. The values of goodness of fit for each of the hydropower dams are $r = 0.95$, $R^2 = 0.96$ for Kanji, $r = 1.00$, $R^2 = 0.99$ for Shiroro, and $r = 0.88$, $R^2 = 0.96$ for Jebba. For the Kanji and Shiroro hydropower dams the log-Pearson type III model gave the best fit, while for Jebba the best fit model was log-normal. These probability distribution models can be used to predict the near future reservoir inflow at the three hydropower dams.

INTRODUCTION

The probability distribution is a hydrological tool most widely used in flood estimation and prediction. The importance of reservoir inflow analysis at any hydropower dam to our daily life makes it imperative that the appropriate probability distribution model be established to determine the discharge into the reservoir. Murray and Larry (2000) stated that the choice of the probability distribution model is almost arbitrary as no physical basis is available to rationalize the use of any particular function. In general, the search for a proper distribution function has been the subject of several studies. Salami (2004) studied the flow along the Asa River and established probability distribution models for the prediction of the annual flow regime. For minimum and maximum flows, log-Pearson type III (LP3) and Gumbel extreme value type I (EVI type I) respectively were recommended. Salami (2002) considered flood levels at four gauging stations along the River Niger, below the Jebba hydropower dam. The maximum and minimum flood level data were fitted with four probability models and compared graphically with the observed data. The EVI type 1 distribution fit the data best and it was used to predict flood levels with return periods of 10, 50 and 100 years. Onoz and Bayazit (1995) dealt with the probability distribution of largest available flood sample with the aim of determining the distribution that best fit the observed flood. The Water Resources Council of the USA conducted a study with the objective of developing a uniform technique of determining flood frequency (Benson, 1968). The work applied the available methods to flood records at 10 stations in various parts of the USA. Record length varied and five methods were used, namely Gamma, EVI, log-Gumbel, log-normal (LN) and LP3 distributions. However, no statistical test was applied to determine the goodness of fit, instead flood discharge for various return periods (2 – 50 years) were obtained from the probability plot and compared with the corresponding values from the five hypothesized distributions. Among these methods, the LP_3 distribution was preferred for common use, and for being capable of fitting skewed data. Cicioni et al. (1973) considered LN, LP3, EVI distributions for the flood data from 108 stations in Italy. Statistical tests such as chi-square (χ^2), Kolmogrov-Smirnov (KS) and probability plot correlation coefficient (ppcc) were applied and the best fitting distribution was found to be LN by the chi-square test while EVI and LP3 were found to be the best by the other test. Beard (1974) estimated the 1000 years floods at 300 stations in the USA with four different models (LN, Gamma, log-Gumbel and LP3). LN and LP3 came close to reproducing the expected exceedences and were concluded to be the best. Vogel et al. (1993) explored the suitability of various models applied to the flood flow data at 38 sites in the southwest USA. The probability distribution models adopted include N, LN, EVI type 1, and LP3, which were compared graphically with the observed data.

Ajayi et al. (2007) estimated the occurrence of flood events and its frequency at the lower Niger basin, Nigeria, using hydrological data including river discharges, runoff records and meteorological data from different gauging stations within the basin. The data collected were subjected to various statistical analyses and plotting position and probability distributions were determined. The results showed that various plotting positions and probability distributions could be used to fit the available discharge records of the River Niger. The EVI distribution was the best of the applied models for peak average reservoir inflow and peak discharge at the River Kaduna (Wuya Gauging Station). The LN distribution best predicted the peak runoff discharge of River Niger (Lokoja Gauging station) and peak discharge at Baro Gauging Station. The predicted models that compared favorably with the observed values are considered the best distribution models.

This study focuses on the evaluation of four methods of probability distribution analysis for the prediction of mean reservoir inflow at the three hydropower dams in Nigeria. This study could serve as a guide to the responsible institutions and dam managers in determining available flow that will generate maximum discharge for hydropower dams and prevent flood waters overtopping the dam, thereby causing subsequent release of a flood wave and averting loss of life and properties (Binnie, 1981). The information can also be a valuable tool for preventive flood forecasting

MATERIAL AND METHODS

Data Collection

The reservoir inflow data were collected from the hydrological unit of the three hydropower stations in Nigeria namely; the Kainji, Shiroro and Jebba hydropower stations. A total of 25 years (1981 - 2005), 16 years (1990 - 2005) and 21 years (1984 - 2005) of inflow data were collected from Kainji, Shiroro and Jebba hydropower stations respectively.

Data Analysis and Evaluation of Probability Distribution Models

The reservoir inflows were ranked according to Weibull's plotting position and the return period was estimated. The reservoir inflow data were evaluated using several probability distribution models to determine the best fit for each of the hydropower stations. The methods include EVI Type I, N, LN, and LP3.

The probability distribution analysis was carried out in accordance to standard procedure (Wilson, 1969; Viessman et al., 1989; Mustapha and Yusuf, 1999). The probability curve fitting was carried out by plotting the reservoir inflow data against the corresponding return period. The mathematical expressions obtained for various probability distributions and the probability curve fitted models are presented in Table 1. The established equations for each model were used to predict values of reservoir inflows and were plotted together with the observed data against the cumulative probability for the purpose of comparison and to study the probability distribution models that best fit the observed reservoir inflow data (Haktanir, 1992). The graphical representations are shown in Figures 1 to 3.

Testing of the probability distribution models

The acceptability and reliability of the probability models were tested by using four statistical tests (goodness of fit tests). The statistical tests include chi-square (χ^2), Fisher's distribution (F), probability plot coefficient of correlation (r), and coefficient of determination (R^2). The statistical tests were carried out in accordance with standard procedure (Chowdhury and Stedinger (1991); Adegboye and Ipinyomi (1995); Bobee et al. (1998); Dibike and Solomatine (1999); Murray and Larry (2000)). The results obtained for chi-square, Fisher's, ppcc (r) and R^2 tests were presented in Tables 2, 3, and 4 respectively.

RESULTS AND DISCUSSION

The reservoir inflow at Kainji hydropower station has ($\chi^2_{cal} / \chi^2_{tab}$), ($F_{tab} / F_{cal}$), r and R^2 values of 0.9500, 0.0160, 0.8500 and 0.9600 respectively for the EVI type I distribution, and ($\chi^2_{cal} / \chi^2_{tab}$), ($F_{tab} / F_{cal}$), r and R^2 values of 0.8900, 0.0110, 0.9200 and 0.9600 respectively for the LP3 distribution. From this result the chi-square test suggests the EVI is best, while other tests suggest LP3 as the best fit model for the reservoir inflow data. The higher value of correlation coefficient (r) for LP3 shows that there is a close linearity between the observed and the predicted reservoir

Table 1. Model equations for the probability distributions.

S/N	Hydropower dams	Probability Distributions	Developed equations
1.	Kainji	Gumbel (EVI) Normal Log – Normal Log – Pearson	$Q_T = 759.050 + 131.150Y_T$ $Q_T = 838.180 + 175.840\ K_T$ $Log\ Q_T = 2.915 + 0.089\ K_T$ $Log\ Q_T = 2.915 + 0.089\ K'_T$
2.	Shiroro	Gumbel (EVI) Normal Log – Normal Log – Pearson	$Q_T = 265.940 + 22.210Y_T$ $Q_T = 278.760 + 28.480\ K_T$ $Log\ Q_T = 2.443 + 0.049\ K_T$ $Log\ Q_T = 2.443 + 0.049\ K'_T$
3.	Jebba	Gumbel (EVI) Normal Log – Normal Log – Pearson	$Q_T = 845.520 + 487.080Y_T$ $Q_T = 1126.520 + 624.460\ K_T$ $Log\ Q_T = 2.948 + 0.115\ K_T$ $Log\ Q_T = 2.948 + 0.115\ K'_T$

Q_T = Expected discharge associated with a particular probability of occurrence
Y_T = Reduced variate
K_T = the value K is listed on the normal distribution table

inflow. Also, based on the graphical comparison (Figure 1) the LP3 distribution model is a better fit than the other probability distribution models. Hence, LP3 is the most appropriate model for the reservoir inflow at Kainji hydropower dam and is thus selected as the best fit model.

The reservoir inflow at Jebba hydropower station (χ^2_{cal} / χ^2_{tab}), (F_{tab} / F_{cal}) r and R^2 values of 2.6200, 0.0140, 0.8800 and 0.9600 respectively for the LN distribution, and (χ^2_{cal} / χ^2_{tab}), (F_{tab} / F_{cal}), r and R^2 values of 8.9000, 0.0310, 1.0300 and 0.8800 respectively for the LP3 distribution. From this result the chi-square test did not satisfy the condition for selection of any model; also the value of the correlation coefficient (r) for LP3 did not satisfy the condition for selection of model. The results of other tests suggest the LN is the best fit model for the reservoir inflow data. Also, based on the graphical comparison (Figure 2) the LN distribution model is a better fit than the other probability distribution models. Hence, the LN is the most appropriate model for the reservoir inflow at Jebba hydropower dam.

The reservoir inflow at Shiroro hydropower station has (χ^2_{cal} / χ^2_{tab}), (F_{tab} / F_{cal}), r and R^2 values of 0.4800, 0.0034, 0.9900 and 0.9960 respectively for Gumbel extreme value (EVI) type I distribution, and (χ^2_{cal} / χ^2_{tab}), (F_{tab} / F_{cal}), r and R^2 value of 0.3600, 0.0030, 1.0000 and 0.9900 respectively for the LP3 distribution. The statistical tests follow the same trend as in the case of the Kainji hydropower dam, i.e. the chi-square test suggests EVI, while other tests suggest LP3 as the best fit model for the reservoir inflow data. The higher value of correlation coefficient (r) for the LP3 also shows that there is a close linearity between the observed and the predicted reservoir inflow. Also, based on the graphical comparison (Figure 3) the LP3 distribution model is a better fit than the other probability distribution models. Hence, LP3 is the most appropriate model for the reservoir inflow at Shiroro hydropower dam and is thus selected as the best fit model.

CONCLUSION

Various probability distribution models were fitted to the reservoir inflow records to evaluate the model that is most appropriate for prediction at the three hydropower stations in Nigeria. Various models were established for each hydropower station and the suitable model was selected based on the goodness of fit tests. The LP3 model was found to be appropriate for both the Kainji and Shiroro hydropower dams, while the LN was found to be appropriate for the Jebba hydropower

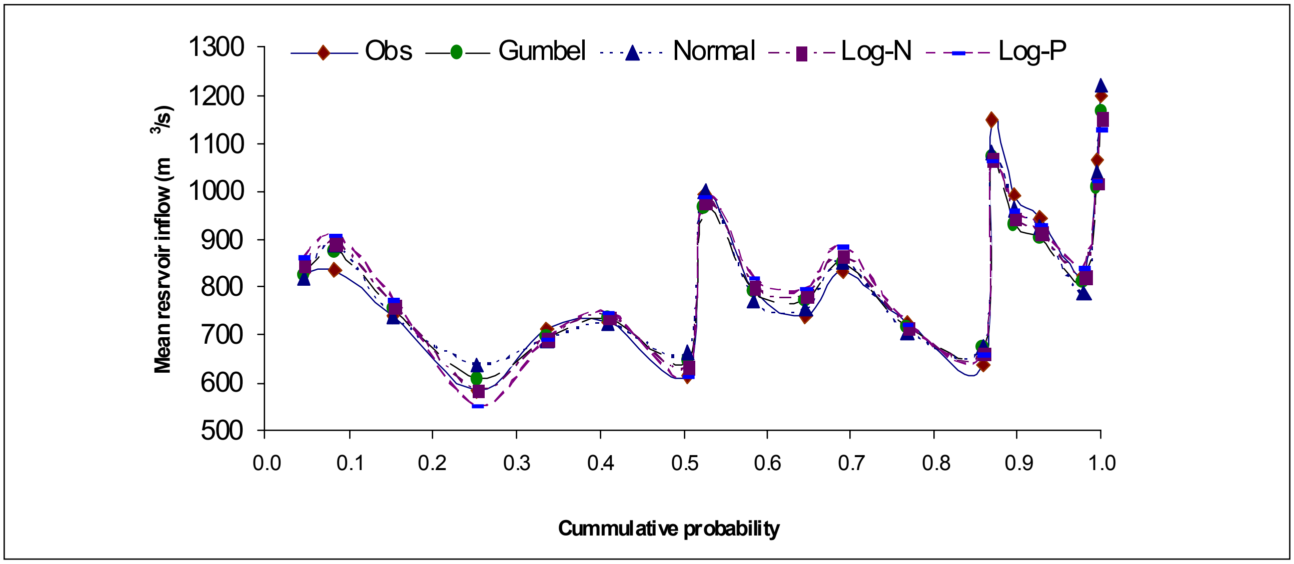

Figure 1. Comparison between observed and predicted flow at Kainji HP dam.

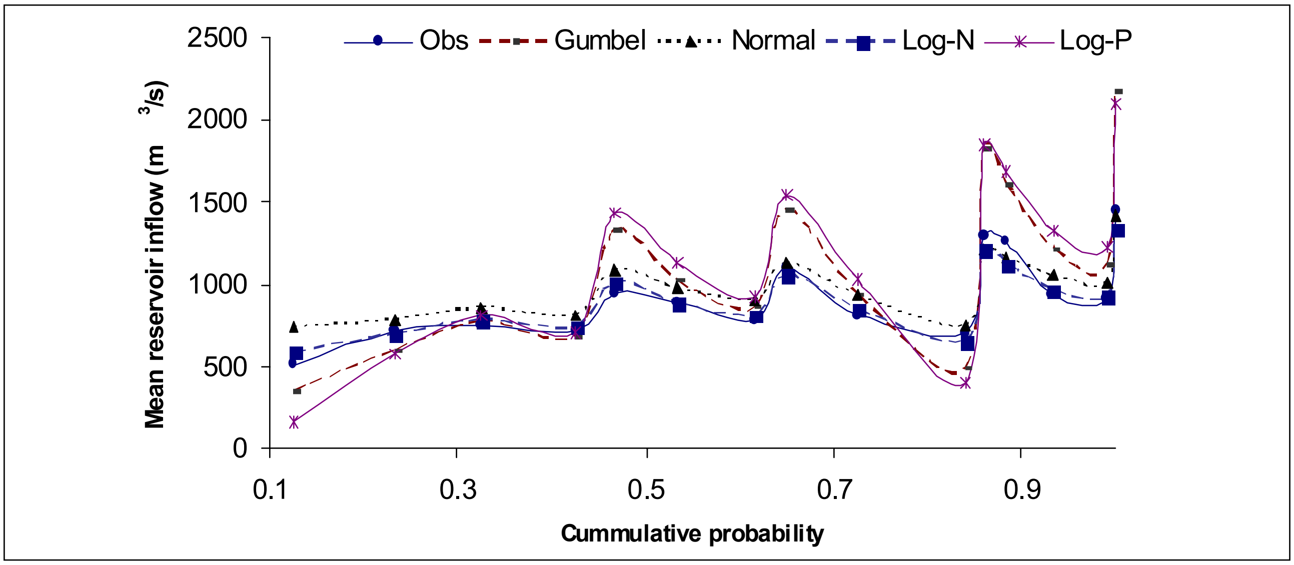

Figure 2. Comparison between observed and predicted flow at Jebba HP dam.

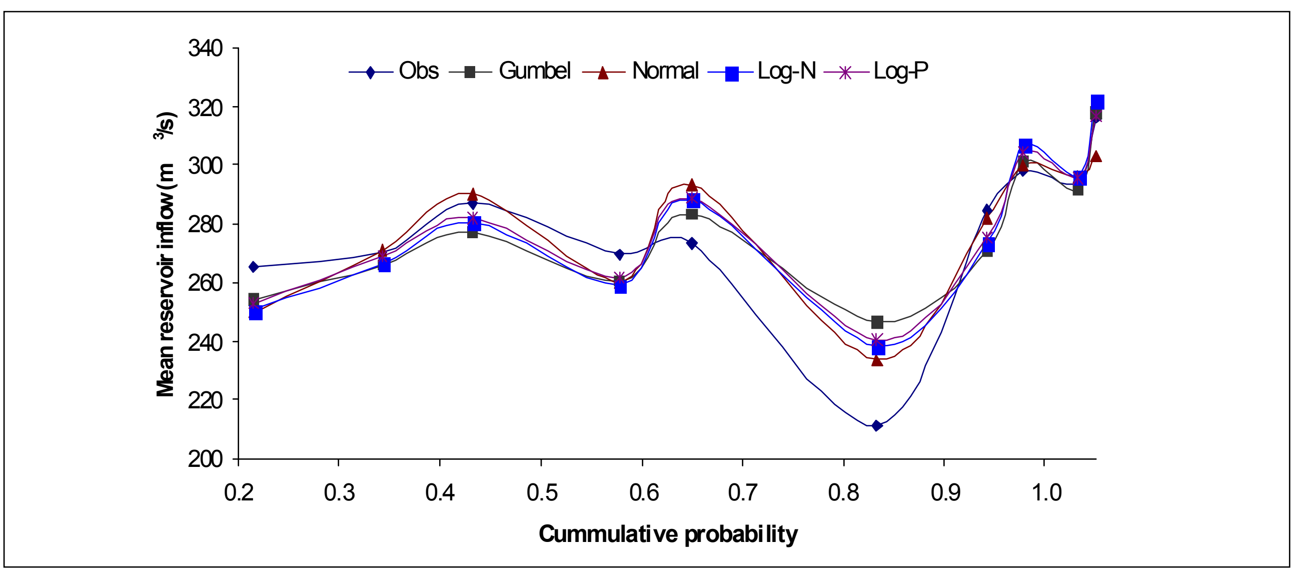

Figure 3. Comparison between observed and predicted flow at Shiroro HP dam.

Table 2. Results for the analysis for Kainji Hydropower dam.

S/N	PDF	Test of Fit			
		χ^2	F	r	R^2
1.	EVI	0.9500	0.0156	0.8500	0.9600
2.	N	1.2300	0.0187	0.8900	0.9400
3.	LN	0.8700	0.0141	0.8600	0.9600
4.	LP_3	0.8900	0.0111	0.9200	0.9600
5.	Probability Curve fitted	1.2000	0.0127	0.9800	0.9500

Table 3. Results for the analysis for Shiroro Hydropower dam.

S/N	PDF	Test of Fit			
		χ^2	F	r	R^2
1.	EVI	0.4791	0.0034	0.9900	0.9900
2.	N	0.3682	0.0025	1.0100	0.9900
3.	LN	0.3953	0.0027	1.0100	0.9900
4.	LP_3	0.3540	0.0025	1.0000	0.9900
5.	Probability Curve fitted	0.4868	0.0034	0.9900	0.9900

Table 4. Results for the analysis for Jebba Hydropower dam.

S/N	PDF	Test of Fit			
		χ^2	F	r	R^2
1.	EVI	45.3815	0.0670	1.9700	0.0100
2.	N	95.5738	0.0820	2.1100	-0.3300
3.	LN	2.6185	0.0140	0.8800	0.9600
4.	LP_3	8.8958	0.0310	1.0300	0.8800
5.	Probability Curve fitted	3.4548	0.0126	0.9700	0.9600

dam. Also, the LP3 model that adequately fits the reservoir inflow at two of the hydropower dams indicates that the inflow data are skewed.

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