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LIQUID ANALOG CIRCUITS FOR LABORATORY SIMULATION OF STEADY-STATE SEEPAGE

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This study proposed a distributed arrangement of a Liquid Analog Model (LAM) for simulating seepage through an embankment dam in the laboratory. Each liquid analog circuit contains two major components of reservoir and friction elements whose parameters are related to those of the prototype dam by appropriate scale factors. Several circuits of the LAM were designed and assembled in the laboratory. To evaluate the capability of the LAM in simulating seepage through the dam, the experiment was conducted in steady-state conditions. The outputs of the LAM were compared to the observed piezometric heads of the dam body and also to the results obtained from a numerical finite difference method (FDM). Experimental measurements revealed that the LAM was a reliable and convenient experimental tool for seepage simulation during steady-state conditions. The ability to provide a visual solution of the seepage partial differential equation is the most advantageous trait of the proposed LAM.

INTRODUCTION

The aim of engineering analysis is not usually limited to mathematical description; rather it is to obtain physical insight into the system. Indeed, prior to the emphasis on engineering science, it could be said that most engineering instructions in general and hydrologic educations in particular take place in the laboratory. In this way, laboratory physical models have long been used and continue to be used to model hydraulic systems and understand their behavior. These models have the advantage of acquiring data under controlled conditions and investigating natural processes under a variety of conditions.

Physical models are either iconic or analog (Singh, 1988). An iconic model is a scaled-down facsimile of the full-scale prototype, and is also referred to as a geometric model. Analog model, on the other hand, measure different physical substances than the prototype (i.e., use another physical system having properties similar to those of prototype), such as flow of electric current which represents the flow of water. An analog model dose not physically resemble the prototype but depends on the correspondence between the symbolic models describing the prototype and the analog system. Iconic models have been applied successfully to certain areas of surface hydrology, such as laboratory simulation of catchment hydrological behavior (e.g., Singh, 1975; Wong and Lim, 2006), as well in the field of subsurface hydrology, like the sand box model which is a reduced scale representation of the natural porous medium (e.g., Koopman et al., 1987; Weil et al., 2009; Berg and Illman, 2012). It is necessary to maintain dimensional conformity in performing a scale operation. Another difficulty in applying scale-model techniques is the difficulty of making accurate measurements at points within the field. To overcome such shortcomings, analog solution appears desirable in fulfilling the primary aims of modeling. (Karplus, 1958).

Several analog models have been employed for surface hydrologic modeling. Likewise, various analog models have been applied to simulate flow of water through porous media, including Hele-Shaw (e.g., Akyuz and Merdum, 2003; Mizumura, 2005) and electrical analog models (e.g., Harder, 1963; Panthulu et al., 2001).

Recently, a Liquid Analog Model (LAM) as a new generation of analog models (Monadjemi, Multipurpose fluid analog computer, U.S. Patent No. 6,223,140, 2001), was applied for laboratory simulation of rainfall-runoff process via a lumped framework (Nourani and Monadjemi, 2006; Nourani et al., 2007). LAM employs liquid (water) in contrast to electricity, which makes it a more convenient laboratory device, especially in hydrologic fields. In the field of surface hydrology and using data from real world watersheds, LAM was applied to laboratory simulation of a conceptual geomorphologic model and the Nash rainfall-runoff model by Nourani and Monadjemi (2006), and Nourani et al. (2007), respectively. The objective of this paper is to employ LAM for subsurface flow analysis through a laboratory experiment. Former studies focused on lumped and time-dependent systems, whereas, as a novel contribution, the current study develops a distributed version of LAM for laboratory simulation of seepage through an embankment dam.

ANALOG MODELING

Analogs are devices with similar cause-and-effect as the prototype but with different properties. For modeling a phenomenon by an analog system, it is not necessary that the analog system and the real system are alike. What is required is the similarity and analogy between the governing equations of two systems. Therefore, the solution of one system can be applied to the other by proper scaling.

Liquid Analog Model (LAM) Components

A liquid analog system consists of at least one circuit and each circuit has two major components: a reservoir element and a friction element. Also, a constant-head overflow element may be employed to apply constant water head for some boundaries. These elements are connected using the friction element (tubes) with an appropriate diameter, so that the flow regime in the tubes remains laminar. The reservoir element is graduated to facilitate the reading of liquid head at any time. Although any kind of liquid can be used in this circuit, water may be chosen because of its accessibility and easy operation. An LAM circuit is presented in Figure 1.

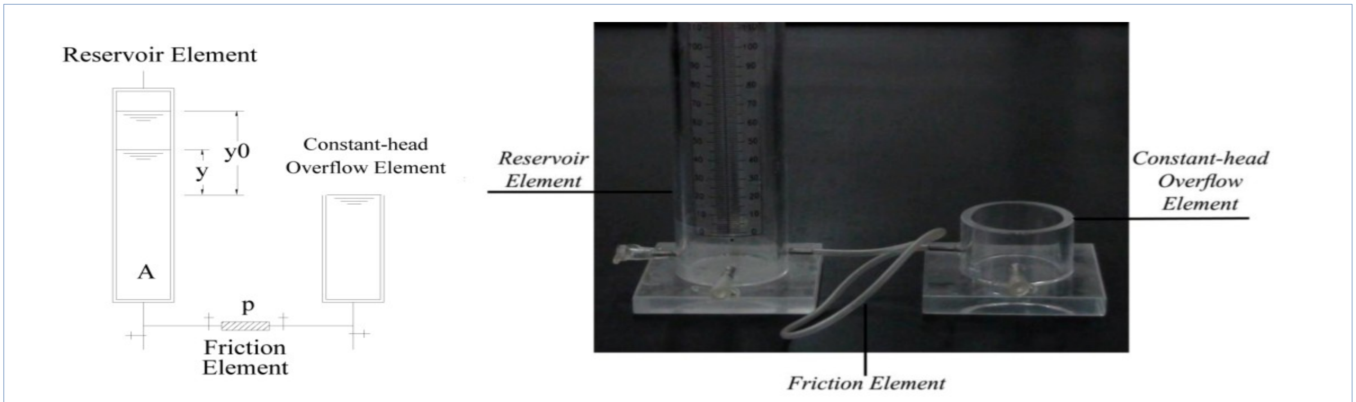

Figure 1. Liquid analog circuit.

To build the friction element in this study, based on the Hagen-Poiseuille law for laminar flow in tubes (Streeter and Wylie, 1988), a tube with appropriate diameter which allows a laminar flow, was employed. According to the Hagen-Poiseuille equation, the head-loss in a circular tube is given by (Streeter and Wylie, 1988):

$$y = \frac{128 L \upsilon Q}{\pi g D_0^4} \tag{1}$$

where, y=head-loss; υ=kinematic viscosity; L=length of the tube; Q=discharge; and D_0 = diameter of the tube. Therefore, the equation describing flow through the developed friction element is written as:

$$Q = \frac{\pi g D_0^4}{128 L \upsilon} y = py \tag{2}$$

From this viewpoint, $\frac{\pi g D_0^4}{128 L \upsilon}$ plays the role of p in the tubular element named friction element with the friction coefficient of p, having the dimensions of $[L^2T^{-1}]$.

Several liquid circuits can be combined in series, parallel, combined series or other complex configurations to construct a system with a known governing equation. In the liquid system if liquid circuits (reservoir and friction elements) are directly connected (Figure 2), the water head in one reservoir will affect the other reservoir; this situation presents a linear system with feedback upstream.

Referring to Figure 2, if the water levels are equal in the reservoir elements, no water will be

Figure 2. One-dimensional distributed LAM circuits.

transferred between reservoirs. However, if an impulse as a water head or discharge is applied to one of the reservoirs, the water will flow in the system. Suppose that at time t the water levels in reservoirs No. 1, No. 0 and No. 2 are y_1, y_0, and y_2, respectively, and the outflows of these reservoirs No. 1 and No. 0 are Q_1 and Q_2, respectively. For reservoirs with the same cross-section area (A), and the same friction coefficient (p) for the friction elements, using Equations (2) and continuity equation for the central reservoir, the following set of equations can be written:

$$Q_1 = p(y_1 - y_0) \tag{3}$$

$$Q_2 = p(y_0 - y_2) \tag{4}$$

$$Q_1 - Q_2 = A\frac{dy_0}{dt} \tag{5}$$

Substitution of Equations (3) and (4) in Equation (5) yields:

$$p(y_1 - y_0) + p(y_2 - y_0) = A\frac{dy_0}{dt} \quad \frac{y_1 - y_0}{\frac{A}{p}} + \frac{y_2 - y_0}{\frac{A}{p}} = \frac{y_0}{dt} \tag{6}$$

This equation represents the feature of a one-dimensional LAM. In addition to the one-dimensional distributed circuits, LAM has the ability to be employed in two and three-dimensional distributions as well. Figure 3 shows a plan of a two-dimensional LAM; by applying Equations (2) and continuity equation in both x and y directions for the central reservoir, the following equation can be written:

$$\frac{y_{x_1} - 2y_0 + y_{x_2}}{\frac{A}{p_x}} + \frac{y_{y_1} - 2y_0 + y_{y_2}}{\frac{A}{p_y}} = \frac{dy_0}{dt} \tag{7}$$

This equation is the governing equation of the two-dimensional distributed LAM in time and space.

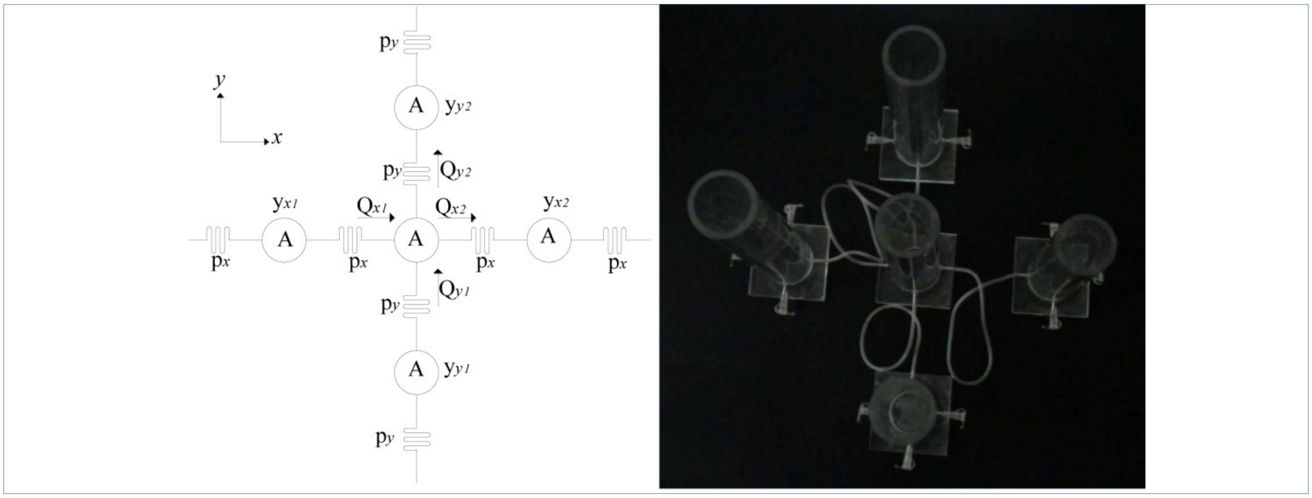

Figure 3. A two-dimensional distributed LAM system.

LAM FOR SEEPAGE ANALYSIS

Basic Differential Equation

The physically – based partial differential equation (PDE) of the mathematical model used in two-dimensional seepage flow through porous media (e.g., embankment dam) can be expressed by Richards' equation (Cooley, 1983). For saturated zones, when hydraulic conductivities are held constant with respect to the *x* and *y* directions, respectively, the upstream water level varies with time and the free water level within interior locations changes more slowly. In this case, Richards' equation can be rewritten as (Bear and Verruijt, 1987):

$$k_x \frac{{}^2 h}{x^2} + k_y \frac{{}^2 h}{y^2} = S_s \frac{h}{T} \tag{8}$$

where *x* and *y* are the horizontal and vertical coordinate directions, respectively; *T* is time in prototype; k_x and k_y are the components of the hydraulic conductivity in the *x* and *y* directions; S_s is the specific storage; *h* is the hydraulic head (h=P+z) and *P* is the pressure head (*z*=elevation).This equation is known as the two-dimensional heat conduction or diffusion equation. In order to solve Equation (8) uniquely, boundary conditions (BCs) must be imposed. the initial and boundary conditions for Equation (8) take the form of (Nourani and Babakhani, 2012):

$$\left\{\begin{array}{c} h = h_B on\, B\; for\, t > 0 \\ q_n = 0\, on\, impervious\, boundaries\; for\, t > 0 \\ h = h_D on\, D\; for\, t = 0 \end{array}\right\} \tag{9}$$

where h_B is the hydraulic head specified on a segment of the exterior boundary *B*, q_n is the specific discharge normal to the boundary segment, and h_D is the initial head distribution in the solution domain *D*.

If time (*T*) in Equation (8) tends to infinite, the right-hand side of the equation would tend to zero, and the most important and the most often encountered fundamental equation of applied physics known as Laplace's equation results. Infinite time, as the point of physical view, means the required time in which hydraulic head in Equation (8) remains constant and the steady-state condition is reached.

Laplace's equation governs those physical fields in which static or steady-state conditions have been achieved.

The finite difference discretization of Equation (8), using the second central difference simulation for the space derivatives, is (Bear, 1979):

$$\frac{h_1 - 2h_0 + h_2}{\frac{S_s}{k_x}(\Delta x)^2} + \frac{h_1 - 2h_0 + h_2}{\frac{S_s}{k_y}(\Delta y)^2} = \frac{\partial h_0}{\partial T} \tag{10}$$

where h_0 is the water head at the central node, h_1 and h_2 are the water heads of adjacent left-hand and right-hand nodes, respectively. Comparing Equations (10) and (7), the analogy between LAM and FDM discretization of the diffusion PDE is revealed. Overall, each parameter and variable in LAM corresponds to a parameter or variable in the diffusion PDE which represents the original field's behavior. In this case, y, t, A/p_x and A/p_y in LAM become analogous to h, T, $(S_s/k_x)(\Delta x)^2$, and $(S_s/k_y)(\Delta y)^2$ in the prototype, respectively. In order to design an LAM system which can fulfill the solution of the problem in a reasonable length of time, it is necessary to select proper scale factors. These scale factors are conversion constants relating the corresponding parameters and variables in the two systems.

Model Scaling and Experimental Set up

To achieve proper scaling of a two-dimensional distributed LAM for laboratory simulation of seepage, it is necessary to consider the governing equations of LAM and seepage phenomenon. Equation (8) shows that there are two independent variables, i.e., head and time. Therefore, two scale factors, head scale and time scale, are necessary. The head scale, l, indicates the relation between hydraulic heads in the two systems (i.e., water heads of the prototype dam in the original field and LAM in the laboratory), and can be expressed as:

$$l = \frac{(y_{Max})_M}{(h_{Max})_P} \tag{11}$$

where h_{Max} is the maximum head in the real system and y_{Max} is the maximum applicable head to the LAM in the laboratory. Here, suffixes P and M refer to variables in the model and prototype, respectively.

The time scale, m, relates the matching time values in the prototype and LAM that means the required time in which steady-state condition is reached in both model and prototype, and can be computed as:

$$m = t_m/T_p \tag{12}$$

Simultaneously, the other scale ratios can be deduced from the equivalency of A/p_x and A/p_y in LAM with$(S_s/k_x)(\Delta x)^2$, and $(S_s/k_y)(\Delta y)^2$ in the prototype, respectively. For this purpose, constant values in both LAM and prototype may be considered analogous, which means A (cross-section area of the reservoir element) in LAM is equivalent to S_s/k in the prototype. Therefore, the reservoir scale ratios, n_x and n_y for both x and y directions can be expressed as:

$$A \sim \frac{S_s}{k} \rightarrow n_x = \frac{A_M}{\left(\frac{S_S}{k_x}\right)_P}, and \quad n_y = \frac{A_M}{\left(\frac{S_S}{k_y}\right)_P} \tag{13}$$

The head and time scales (l and m) are dimensionless, but the coefficient n takes the dimensions of $[L^4T^1]$.

Finally, based on the obtained scales and the desired discretization of the study domain with finite increments of Δx and Δy in the x and y directions, respectively, the friction coefficient, p, for LAM can be deduced as:

$$p_x = \frac{n_x}{m}\frac{1}{(\Delta x)^2}, \quad and \quad p_y = \frac{n_y}{m}\frac{1}{(\Delta y)^2} \tag{14}$$

where p is the friction coefficient with the dimensions of $[L^2T^1]$. From a physical point of view, the reciprocal of the friction coefficient, $1/p$, represents the resistance of flow in one specific direction (x or y). The friction element, therefore, simulates the energy-dissipating or damping characteristics of the original field; while, in the finite difference approach, the space intervals (Δx or Δy) represent this characteristic. Therefore, according to the analogous equations of Equations (7) and (10), the reciprocal of friction coefficient ($1/p$) in each direction must be proportional to the square of space intervals across the same direction.

Generally, the behavior of the system to be simulated (i.e., flow through porous media in saturated zone) is described by the diffusion PDE, and LAM is found whose equations are similar in form. Based on this analogy, to predict the hydraulic heads in the original field at a given point, it is only necessary to determine or measure the height of water in the LAM's reservoir element at the corresponding point. Then, the measured data in the laboratory are converted to the original field scales using corresponding scale factors.

The last step of model scaling is to impose the boundary conditions (BCs) of prototype to LAM. The applied flow BC in the seepage problem may be Dirichlet (imposed head), Neumann (imposed flux), or their combined types. To apply the Dirichlet BC to LAM, the head would be imposed on the system through the constant-head reservoir elements. For the Neumann BC, it is the flux which could be imposed to the desired reservoir elements via a suitable electric pump. At the impervious boundaries, the reservoir elements would not be connected to any other element, as is with the existing condition in nature.

As the final step of the model design, the length of friction element is to be calculated. For this purpose, diameter of tube (D_o) based on the laboratory facilities is considered. The values of gravitational acceleration (g) and kinematic viscosity (υ) are constant; hence, the length of friction element according to Equation (2) is computed for both x and y directions:

$$L_x = \frac{\pi g D_0^4}{128 p_x \upsilon}, \quad and \quad L_y = \frac{\pi g D_0^4}{128 p_y \upsilon} \tag{15}$$

where L_x and L_y are the lengths of friction elements, in the x and y directions, respectively.

In order to scale and design the LAM circuits for simulating seepage phenomenon in the laboratory, the following procedure is followed. First, a proper reservoir element, based on the laboratory equipment, is considered and the head scale (l) is obtained according to the maximum water head in the prototype, h_{max}, (e.g., maximum water head in the dam reservoir) and Equation (11). Second, a reasonable time scale (m) is chosen. Next, using Equation (13) the coefficient of n is computed. Finally, the flow domain (earth dam body in this study) is divided by gridlines in which the x and y axes are divided into intervals of Δx and Δy, respectively; and the friction coefficient (p) is determined by Equation (14). In the last step, allocating suitable values to D_0 (according to the laboratory facilities), g, and υ, the lengths of the friction elements in both x and y directions (L_x and L_y) are computed using Equation (15).

Calibration of individual LAM circuits with different friction elements, in order to verify the governing equation of an LAM circuit, is strongly recommended. In this way, an LAM circuit for each length of friction element should be tested prior its connection to other circuits. By comparing the observed and theoretical results of an individual circuit, the value of kinematic viscosity (υ), which is strongly sensitive to the laboratory temperature, can be also determined.

RESULTS AND DISCUSSION

In this section, the capability of the proposed LAM was examined by simulation of the Sattarkhan embankment dam's seepage phenomenon under steady-state condition in the laboratory.

Sattarkhan Embankment Dam and Data

Sattarkhan embankment dam is a reservoir dam on the Ahar River, in the East-Azerbaijan province, Iran. The height of the dam is 59 m above the alluvial deposit layer and its crest length is 340 m. The reservoir capacity (while the normal water level is 1451 m above the mean sea level) is 131.5 million m^3.

At the four cross sections of the dam several electrical piezometers have been installed to monitor water heads through the dam. The daily water levels in the piezometers and dam's reservoir have been monitored in the first month of dam's reservoir initial filling (from April 1998 to May 1998) and considered in this study. Based on the East-Azerbaijan Regional Water Corp report (1998), the values of the hydraulic conductivity in the x and y directions, and specific storage (k_x, k_y and S_s) are $5\text{x}10^{-8}$ m/s, $5\text{x}10^{-7}$ m/s, and 10^{-4} 1/m, respectively. Also, the statistics of the observed heads in section No. 2 have been tabulated in Table 1. Figure 4 shows the piezometer positions of section No. 2. Since most head losses occur through the dam's core, most instrument installations and monitoring programs are concentrated in this part of the dam. Consequently, in this study the core of Sattarkhan embankment dam was considered as the study area and computational domain.

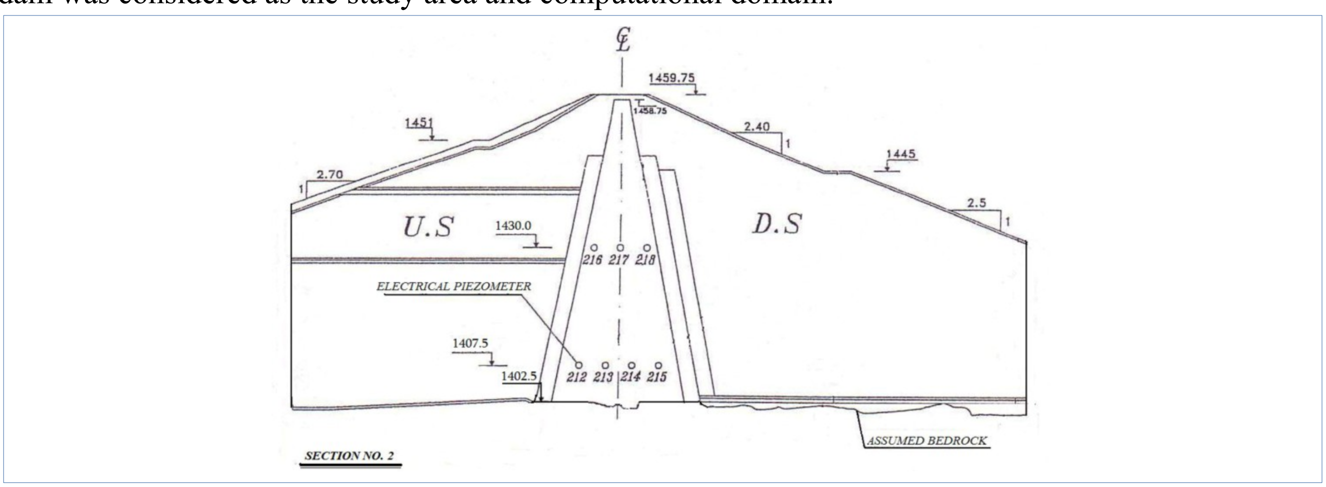

Figure 4. Piezometer positions of section No. 2, Sattarkhan embankment dam.

Table 1. Statistics of the observed heads in section No. 2.

	Reservoir	Piezometer No.			
		212	213	214	215
Max. Water Level (m)*	20	19.3	17.65	13.1	10.1
min. Water Level (m)*	19.3	5.3	5.1	5.1	5
Mean Water Level (m)*	19.63	17.06	15.33	11.1	8.6
Standard Deviation σ(m)	0.225	4.81	4.27	2.8	3.53

*Elevations are from the assumed bedrock

Model Design

To design an LAM system for laboratory simulation of Sattarkhan embankment dam's seepage process, the flow domain was divided by gridlines as shown in Figure 5. Like FDM, one of the advantages of LAM is its ability to deal with different spatial intervals, which means the distance between gridlines does not need to be constant throughout the study domain. In order to represent this ability of LAM, also to set a reservoir element on each grid of the upstream and downstream boundaries, the study domain through the x axis was divided into a grid with variable spacing (see Figure 5), whereas the intervals through the y axis were maintained constant(Δy=5m).

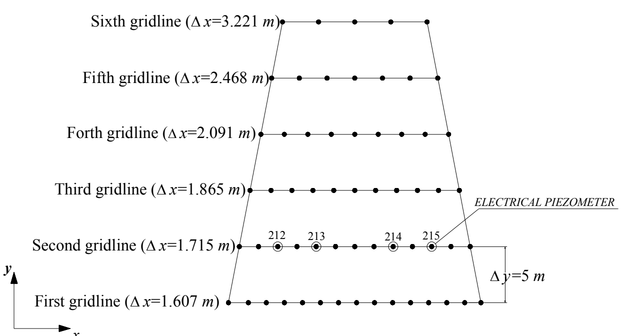

Figure 5. Gridlines of the computational domain.

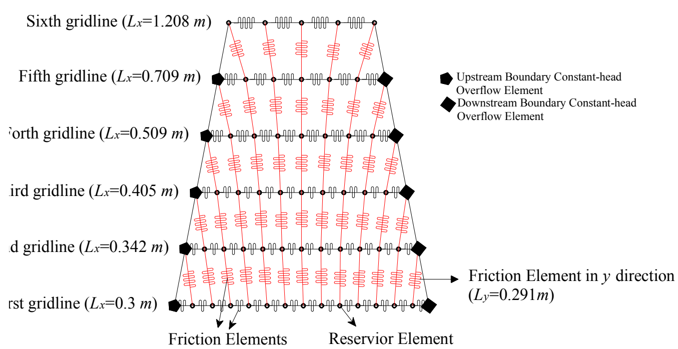

Figure 6. Layout of the experimental facility.

The maximum height of the reservoir elements considered for the LAM set up was 0.3 m, fabricated from 0.04 m diameter Plexiglas cylinders with appropriate gauging to ease the reading of water levels in each grid (see Figure 1 to Figure 3). Therefore, according to the maximum value of the observed water head in the simulation period (20 m, see Table 1) and using Equation (11), the head scale was calculated as:

$$l = \frac{(y_{Max})_M}{(h_{Max})_P} = \frac{0.3}{20} = 0.015 \tag{16}$$

On the other hand, in order to calculate the time scale, time duration of a week (7 days) after the initial filling of the dam's reservoir was considered for simulation in the laboratory, while optionally a 5.5 hour duration was considered a satisfactory period of time for the experiment; therefore using Equation (12), the time scale was computed as:

$$m = \frac{t_M}{T_p} = \frac{5.5}{7 \times 24} = 0.033 \tag{17}$$

In the next step, according to Equation (13) the reservoir scale ratios in both x and y directions (n_x and n_y) were computed based on the constant values of cross-section area of the reservoir elements (A), specific storage (S_s), and the hydraulic conductivities in the x and y directions(k_x and k_y):

$$n_x = \frac{A_M}{\left(\frac{S_s}{k_x}\right)_p} = \frac{\pi \times \left(\frac{0.04^2}{4}\right)}{\frac{10^{-4}}{5 \times 10^{-5}}} = 6.28 \times 10^{-7} \text{ and } n_y = \frac{A_M}{\left(\frac{S_s}{k_y}\right)_p} = \frac{\pi \times \left(\frac{0.04^2}{4}\right)}{\frac{10^{-4}}{5 \times 10^{-7}}} = 6.28 \times 10^{-5} \quad \left(\frac{m^4}{s}\right) \tag{18}$$

Thereafter, according to Equation (14) and based on the space intervals in both x and y directions (Δx and Δy), the friction coefficients in x and y directions(p_x and p_y) were calculated. The length of each friction element (L_x and L_y) in LAM was corresponding to the related space interval in FDM. According to Equation (15) in order to compute the length of each friction element, first a proper diameter of tube which allows flow to remain laminar is to be chosen. For this purpose a tube with an inside diameter of 1.8 mm was considered. Then, using Equation (15) and the calculated values of p_x and p_y for each space interval, the length of the related friction element was calculated (Table 2). Finally, the LAM circuits were assembled and connected to each other through the related friction elements. For the sake of conciseness, the features of the designed LAM are tabulated in Table 2. Also, the layout of the experimental facility to model Sattarkhan embankment dam's seepage process is shown in Figure 6.

Table 2. Characteristics of the designed LAM

Gridlines in x direction	1st	2nd	3rd	4th	5th	6th
No. of reservoir elements	15	13	11	9	7	5
Space intervals in x direction (Δx) (m)*	1.607	1.715	1.865	2.091	2.468	3.2
P_x(m^2/s)**	7.37×10^{-6}	6.47×10^{-6}	5.47×10^{-6}	4.35×10^{-6}	3.12×10^{-6}	1.83×10^{-6}
L_x (m)*** (D_0=1.8mm)	0.3	0.342	0.405	0.509	0.709	1.208

*Δy =5 m constant for all gridlines in y direction

**p_y=7.61×10^{-6} (m^2/s) constant for all gridlines in y direction

***L_y=0.291 m constant for all gridlines in y direction

Model Verification

In this section, the ability of LAM to simulate the water table and seepage face under steady-state condition was analyzed. To verify LAM, results were compared to the observed data of piezometers, and also to the obtained results of FDM.

It is apparent that if steady-state conditions exist, that is, if none of excitations is a function of time, or if sufficient time has elapsed since any pervious change in excitation, the $\partial h / \partial T$ term in the diffusion equation (Equation 8) vanishes, and $\nabla^2 h$ is equal to zero. Laplace's equation may, therefore, be considered as a special or degenerate case of the diffusion equation. This implies that field problems governed by the diffusion equation are described by Laplace's equation if static or steady-state conditions exist. In order to simulate the steady-state condition in LAM, constant-head overflow elements were designed to impose a constant head BC at the upstream end of the system (as Dirichlet BC). To model 20 m upstream head of Sattarkhan embankment dam in the laboratory, the height of the constant-head reservoir elements was calculated based on the head scale (l) as:

$$(y_M)_{upstream} = l \times (h_P)_{upstream} = 0.015 \times 20 = 0.3\ m \tag{19}$$

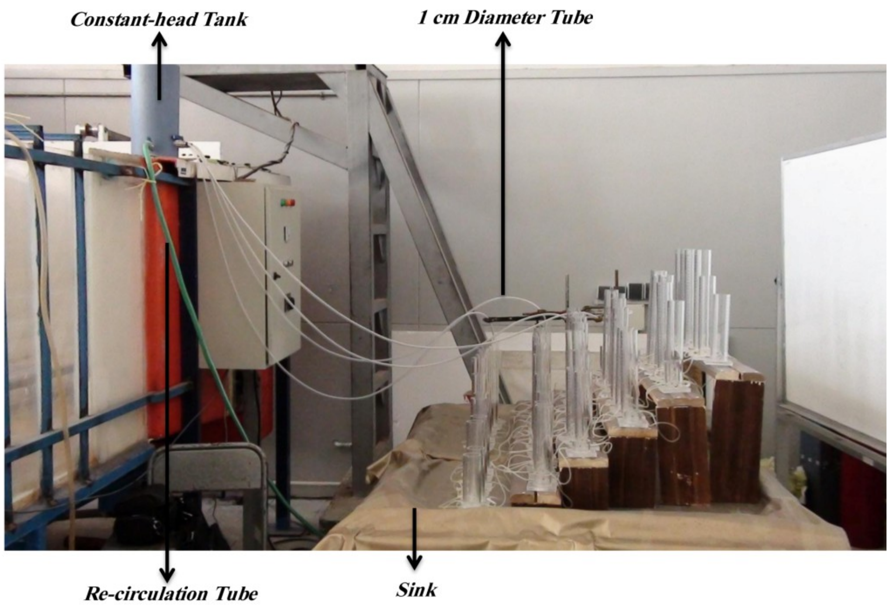

Figure 7 Constructed LAM in the laboratory

To control the entry head of water, a constant-head tank was employed which supplied water through five 1 cm diameter tubes to the five upstream constant-head overflow elements (upstream BC). To save water, the overflow from the constant-head elements was directed to the sink to be re-circulated to the constant-head tank and then again to the upstream elements (see Figure 7). On the other hand at the downstream external boundary of the embankment dam along the seepage face, water emerges from the porous medium into the external space. A seepage face is an external boundary of the saturated zone where flux is directed outward and there is atmospheric pressure along that boundary. The atmospheric pressure was maintained for all the nodes along the seepage face and they were treated as Dirichlet nodes with the prescribed zero pressure. Therefore at the seepage face boundaries of the designed LAM, the height of the downstream constant-head overflow elements would be proportional to the elevation of the analogous nodes in the dam body, which means:

$$(y_M)_{downstream} = l \times (z_P)_{downstream} \tag{20}$$

Where $(y_M)_{downstream}$ is the height of the constant-head overflow elements at the downstream boundaries of LAM, whereas$(z_P)_{downstream}$ is the elevation of the analogous nodes in the dam body. Indeed the reservoir elements in which water overflows from would be on the seepage face. The heights of the constant-head overflow reservoir elements at the LAM's downstream boundaries are summarized in Table 3.

Table 3. Characteristics of LAM's constant-head overflow elements at downstream BCs.

Elevation of the node at the downstream boundary of dam (in prototype) (Z_p) (m)	0	5	10	15	20
Height of the constant-head overflow elements at the downstream boundary of LAM (in model) $(y_M)_{\text{downstream}}$ (m)	0	0.075	0.15	0.225	0.3

A steady-state condition was completely reached when the elevation of the water tables in the LAM reservoirs remained constant over time. Once this condition was reached in the laboratory, the water head of each reservoir element was read and de-scaled to obtain the results in the prototype scale. Due to the limitation of the installed piezometers in the dam body, only the water heads of the second gridline's reservoir elements of the LAM could be comparable with the observed heads in the piezometers. However, a complete comparison was performed between the results of LAM and FDM to assess the ability of the designed LAM and also to investigate the analogy between FDM and LAM.

The obtained results were evaluated according to the Root Mean Squared Error (*RMSE*) and Determination Coefficient (R^2) criteria (Nourani and Babakhani, 2012). Obviously the closer *RMSE* to 0 and R^2 to 1, the better would be the model.

Accompanying the outputs of LAM (hydraulic heads of the reservoir elements placed on different gridlines), the obtained results of FDM are presented in Figs. 9 a-f and the values of *RMSE* and R^2 are listed in Table 4 which show a good agreement between experimental and numerical results and also observed data of the second gridline of the dam.

As shown by Figure 8, although results of the experiment closely fit the numerical results in general, there is a bit of overestimation in the LAM results compared to the FDM where this overestimation decreases from upstream toward downstream. This trend is also the same when the results of LAM are compared to the observed data in the second gridline (Figure 8 b). The reason can be linked to the viscosity issue. Since the experiment was conducted in summer, the viscosity of water during the experiment decreased; noting this fact that according to the Hagen-Poiseuille law (Equation 1), the head loss depends upon the fluid viscosity, which can decrease due to temperature in the laboratory. Subsequently, measured hydraulic heads were overestimated. However, viscosity changes in itself is not the only criterion for the error involved. Minor losses which are not considered in the formulization can also effect the overall accuracy. It is interesting to note that the head losses of the upper gridlines do not decrease noticeably toward downstream grids (Figures 9 d, e, and f). Concerning the impacts of viscosity changes and minor losses on the measured heads, it seems that the viscosity changes influenced the water heads of reservoir elements at all gridlines, and led them to be a bit overestimated. Nevertheless, since spatial discretizations increased in the upper gridlines, which led to the increase of the friction elements' length, the minor losses decreased.

On the other hand, the fewer reservoir elements on the gridlines, the less minor losses. Generally, if there are no temperature fluctuations during the experiment, the precision of LAM not only on the upper gridlines but also at the upstream nodes would be improved. Meanwhile, Figure 8 indicates the capability of LAM to simulate the seepage face. At the exit point of the second gridline (Figure 8 b), water overflows from the constant-head overflow element which means this grid is on the seepage face. However, at the exit point of the third gridline, the hydraulic head is not high enough for the water to be able to overflow. Hence, the pressure head at this point is negative, and the grid is just above the seepage face. These results are in agreement with the numerical results and observed data.

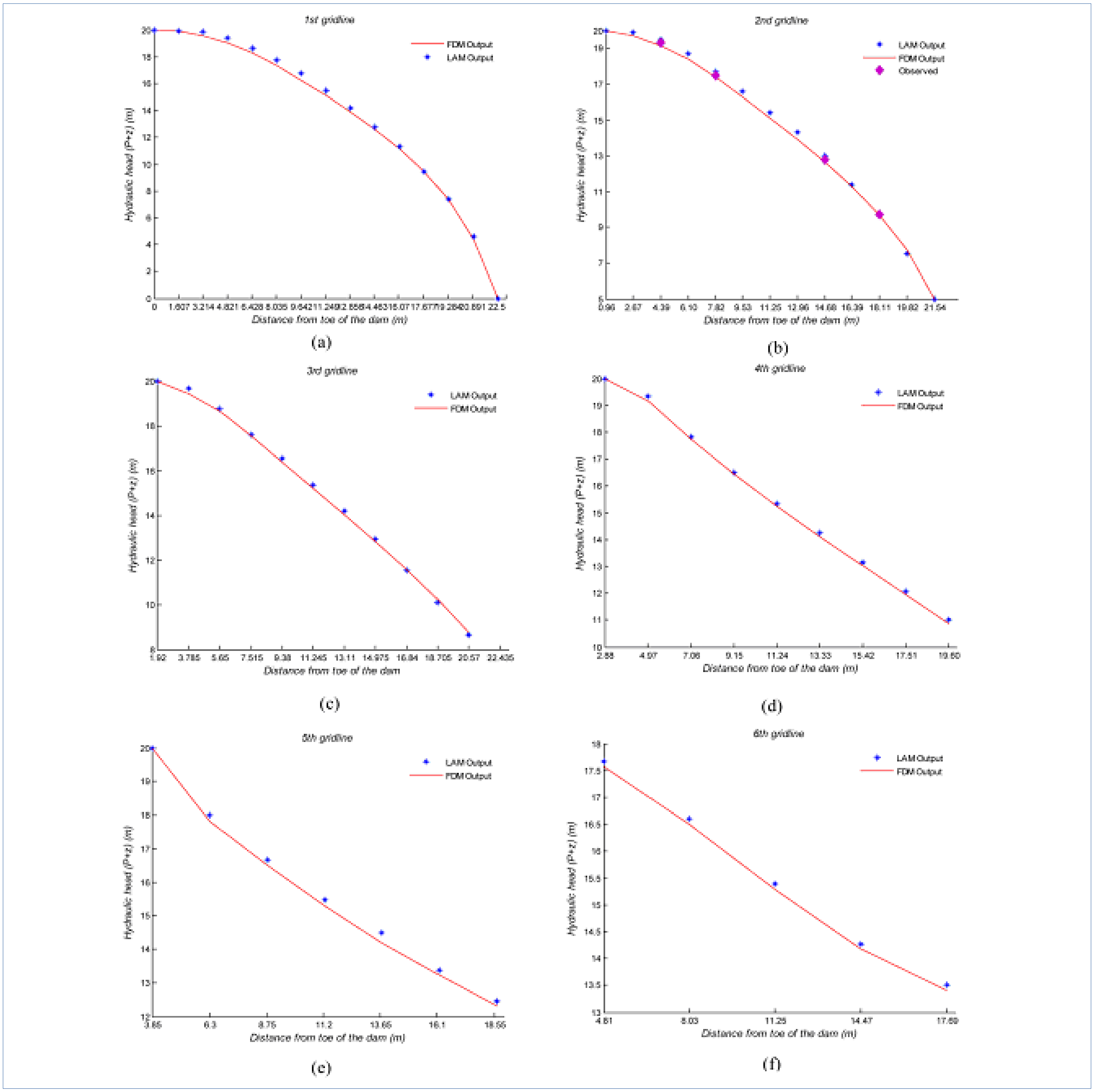

Figure 8. Comparison of the hydraulic heads of (a) first, (b) second, (c) third, (d) forth, (e) fifth, and (f) sixth gridline in the steady-state condition.

Table 4. Performance of LAM compared to FDM and the observed data.

Gridline in x direction	FDM						Observed
	1st	2nd	3rd	4th	5th	6th	2nd
RMSE (m)	0.271	0.262	0.139	0.115	0.184	0.096	0.174
R^2	0.9979	0.9968	0.9985	0.9985	0.9945	0.9959	0.9979

In spite of the errors involved in the experimentation, the overall results show a high efficiency of LAM in simulation of water head through Sattarkhan embankment dam under steady-state condition. The experimental measured data uncover the ability of the designed LAM in solving groundwater (seepage) PDE as well as providing a physically sound practical basis for analyzing flow through porous media. Indeed, the ability to provide visual solution of the PDE is the most advantageous trait of LAM.

CONCLUSIONS

Seepage analysis plays an important role in the designs of hydraulic, environmental and civil engineering. Especially for hydraulic structures such as embankments, the problems of seepage failure are significantly affected by seepage. In this paper, seepage process through Sattarkhan embankment dam was simulated by LAM. Experimental measurements reveal that LAM, as a new laboratory approach, is a reliable and convenient experimental tool, which can provide visual solution of the seepage PDE.

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